Jacob de Gelder (1765-1848) and his way of teaching cubic equations

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Abstract

Jacob de Gelder was a mathematics teacher who wrote many textbooks, inspired by his teaching experience. In his books you can almost hear him talking to, discussing with or questioning his pupils. He combined two styles, mathematical correctness and emphatic expression just like he did in his classroom.

Keywords: cubic equations; Jacob De Gelder; didactics; Socratic approach

Introduction

Jacob de Gelder (1765-1848) was a remarkable mathematics teacher with an intriguing career in the Netherlands in pre- and post-Napoleonic times. He wrote textbooks about mathematics for his pupils and books about mathematics and didactics for teachers and mathematicians.

These works are still of interest, as I will illustrate by the way in which De Gelder taught the solution of cubic equations. To modern eyes, the mathematics of solving cubic equations algebraically may seem old-fashioned, but his insights in didactics and pedagogics are still worth remembering.

Fig. 1. Jacob de Gelder, portrait by Berghaus, collection Universiteit Leiden - Academisch Historisch Museum

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Biography

De Gelder wrote in Dutch for a wide audience. Nowadays his works are studied by those who take an interest in Dutch history of mathematics education. Books and articles about De Gelder have been written in Dutch (van der Aa, 1862; Beckers, 1996b, 1996c; Smid, 1997), and in English (Beckers, 1996a, 2000). The following biography is mainly based upon these articles and books unless indicated otherwise.

Jacob de Gelder was born in 1765 near Rotterdam in a lower-middle-class family. He went to a so-called French school, which served as a good education and also prepared for a career in, for example, business, administration or engineering (Kruger, 2014). Latin was not taught, so unlike the Latin school, a French school did not give access to university. After finishing this school De Gelder worked as an assistant teacher at the same school. A few years later, he opened his own school in Rotterdam and recruited pupils who prepared for a naval career. In 1793 he published his first book *Grondbeginsels der Cijfferkunst* [Principles of Arithmetic] (Gelder 1793).

After the upheaval of the French Revolution, things changed in the Netherlands too. In 1795, the Batavian Republic replaced the old government and many institutions were reformed. Between 1806 and 1810, Louis Napoleon, brother of the emperor Napoleon, was king. After 1810 the Netherlands were annexed by Napoleon. In 1815 the Netherlands became an independent kingdom, including Belgium and Luxemburg. In 1830 Belgium separated and became independent, but it took the Dutch king nine years to accept that fact. Jacob de Gelder witnessed all these political events.

De Gelder had to close his school in 1795 and had difficulty in getting employment elsewhere. Thanks to his friend, the Amsterdam professor Van Swinden, De Gelder was invited to work for the Dutch triangulation project, mapping the Netherlands. Possibilities were also provided by cultural and scientific societies, which were very popular in those days. Some societies owned prominent buildings where their members met, stimulated each other in reading and writing articles and discussed current topics in science, art, and politics. De Gelder taught mathematics to the members of the society Diligentia in The Hague and their children. These lessons resulted in a new book on algebra, *Wiskundige lessen* [Mathematical lectures] (Gelder, 1808). During the reign of King Louis Napoleon, De Gelder took care of the mathematics education of his royal staff. In that period, De Gelder was offered a position at a newly planned Military Academy, but this project was not realized until a Military Academy was founded in 1814 by King William I, with De Gelder as professor of mathematics.

At first, the commander of the Military Academy, Voet, praised the didactics of De Gelder, but later he changed his mind. Conflicts arose about how much mathematics should be taught to non-mathematicians and how rigorous. Eventually, in 1819, De Gelder was dismissed. Later in his life, De Gelder was also embroiled in other conflicts, mainly concerning his ideas about teaching mathematics. Although De Gelder was not explicit in his books in the early years, we now know that he wanted his pupils to start reasoning for themselves. For a modern reader this idea might be common, but in those years, it was innovative to write that you want pupils to learn why things are true or not true.

An interesting difference of opinion arose in 1821 when De Gelder was asked to teach mathematics at the Leiden Latin school. In Latin schools, mainly Greek and Latin were taught, and De Gelder ran into conflict with his principal, Bosse, concerning the question of how much mathematics should be taught. According to De Gelder, mathematics was necessary, but Bosse completely disagreed. According to him, De Gelder was too ambitious and too demanding, reaching far beyond what his pupils needed. Beckers (1996a) puts it this way:

> De Gelder thought that mathematics was indispensable to anyone: to him, mathematics gave gateway to a way of thinking which prevented one from making mistakes. Mathematics gave certainty, mathematics provided a way of dealing with problems, which was far more powerful than any other science or knowledge. To him it was inconceivable not to educate people in mathematics: for De Gelder mathematics had a propaedeutic function in the education of people.
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> To Bosse and Voet however, mathematics was a more or less convenient way of solving a limited number of problems. To them, mathematics was important for people who were able to put it to immediate practical use. Mathematically proven truth was not worth more to them than any other kind of truth.

These conflicts show that De Gelder possessed a strong opinion on mathematical education. Although he lost his job in these conflicts, they did not destroy his career. In 1819, he became a professor by special appointment at Leiden University and in 1824 he was appointed a regular university professor of mathematics there. In 1826, the Dutch government decided to require the teaching of mathematics at all Latin schools and the books of De Gelder were recommended. Finally, his son started the Pedagogium, a pre-university boarding school, in Leiden. Thanks to the sketches and the diary of one of his son's pupils, we have an idea of how pupils experienced life at such a school (Bervoets, 1985; Bervoets & Chamuleau, 1985). Jacob de Gelder died in 1848. He had seen many changes in his country and with regard to mathematics education, he wanted things to change and was one of the actors.

Fig. 2. Jacob de Gelder around 1835-1840 by Alexander Verhuell, collection: Gelders Archief

How to teach mathematics

De Gelder wrote his ideas about education and mathematics in his book *Verhandeling over het verband en den zamenhang der zedelijke en natuurlijke wetenschappen* [Treatise on the relationship and the coherence of the moral and natural sciences] (Gelder, 1826). The fifth chapter deals with his way of teaching. De Gelder wrote that he was teaching young pupils aged 12 to 16 and warns us not to overload these young children with too much homework. He wrote that some teachers complained that these young children cannot learn anything at all. De Gelder replied that these teachers forgot the playful and inventive monkeyshines of their pupils in which they display their shrewdness and prove that they do learn a lot (but outside the official curriculum).

De Gelder distinguished between two ways of teaching. One is the perfect demonstration by a well-prepared teacher who knows everything and explains all details in the right order. The pupils hear him talking and feel that the teacher is right, but would not know how to find a mistake. In this situation, the pupil accepts the teacher's reasoning because the teacher is their master. Pupils can repeat what their teachers have said, but do not know how to say it in their own words. Pupils cannot make small modifications because the reasoning is not their own reasoning. The other way is the Socratic discussion where the teacher keeps on asking questions, forcing the pupils to think for themselves, to internalize the reasoning, to find words of their own. When a pupil is really sure about the logic, when he masters the topic, he is able to explain things himself. Now, a pupil is in a different position during a lecture, because he is able to detect (small) errors during a not-so-perfect demonstration by the teacher, or even correct these errors.

Critics may say that De Gelder presented examples of successes only, suspecting that he quoted his best-performing pupils and that he made such small steps during the Socratic discussion that he was almost feeding them the right answers. Indeed, his examples are too good to be true according to modern standards, however, one cannot say that De Gelder is hiding his belief in the power of the Socratic way of discussion.

Solving cubic equations

Solving cubic equations is a good example to demonstrate De Gelder's way of teaching. From the seventeenth through the early twentieth centuries, the algebraic solution of polynomial equations of degree 3 and 4 was treated extensively in many textbooks; see for example *De geheele Mathesis of Wiskonst* [The whole mathematics] (Graaf, 1676), *Wiskundige lessen* [Mathematical lectures] (Gelder, 1808), *De theorie en de oplossing van hoogere magtsvergelijkingen* [Theorie and solution of higher degree equations] (Ven, 1864) *Lobatto's lessen over de Hoogere Algebra* [Lobatto's lessons on advanced algebra] (Rahusen, 1892) or *Lessen over de Hoogere Algebra* [Lessons on advanced algebra] (Schuh, 1924).

In *Beginselen Der Stelkunst* [Fundamentals of algebra] (De Gelder 1819), Jacob de Gelder starts chapter 9 of Section 2 with an introduction on the Italian mathematician Cardano (1501–1576). He explains Cardano's approach for solving the cubic equation in modern notation and derives the single real solution, which Cardano found. De Gelder then refers to a previous chapter in which he states that a cube root of any (non-zero) number always has three possible (real or complex) values. By doing so, he raises the question concerning the remaining two solutions of a cubic equation with three real roots (De Gelder uses the word "bestaanbaar"). Patiently he explains how to construct the other two solutions of the general cubic equation. He urges the reader to try this himself. He stimulates the reader to write down the general case so that the reader is really aware that the construction is valid in general. This example makes clear what De Gelder is up to: he wants his students to accomplish true general mathematical reasoning without numerical examples. One can imagine that this way of doing mathematics was much more demanding for his pupils and colleagues than simply doing numerical examples and exercises.

De Gelder continues by asking questions about the existence of real solutions and wants his readers and pupils to investigate the mathematical expressions of each intermediate step. He shows that mathematical reasoning will tell you under which circumstances all three solutions are real, and also when there is only one real solution. He does not present a prescription but tries to explain his reasoning. One of the particularities of the cubic equation is that attempts to simplify its solution algebraically might result in a new cubic equation which means that you get stuck in an endless loop without getting any further. De Gelder explains that this is sometimes unavoidable. See below for an example.

Audience

De Gelder wrote a number of textbooks and revised some of them. In the preface of *Wiskundige Lessen I* (Gelder, 1808, pp vii) one can read that this book was meant for young pupils, aged 13 or 14 years or older. According to the title page, the book had to be used by teachers in the classroom. De Gelder often wrote sentences like "the pupil should …", and this book is no exception. In the preface of *Wiskundige Lessen II* (Gelder, 1809, pp vi) one can read that this book was meant for pupils, aged 15 years or older. In the preface of *Beginselen der Stelkunst* (Gelder, 1836, pp v), the book is addressed to the young people starting to learn mathematics.

Language

It is characteristic for De Gelder and his contemporaries to use emotional expressions such as "zwarigheid" and "voor altijd moet wanhopen". In English, one would say "severe difficulty" and "despair forever" or even "despond". Here you see that De Gelder is talking to pupils, non-mathematicians, young or old, who might get stuck in their attempts to learn mathematics. De Gelder's textbooks reflect his preference for Socratic reasoning, consisting of a dialogue with questions and answers. In my opinion, this chapter on the cubic equation is a good example of how to guide pupils through this topic.

Mathematics

We will now look at an example of De Gelder's explanation of what he calls the severe difficulty and what he calls the unavoidable endless loop. In *Wiskundige Lessen* (Gelder, 1828), De Gelder introduces the equation $x^3 + px + q = 0$. He mentions that, according to Cardano's formula, $x = \sqrt[3]{-\tfrac{1}{2}q + \sqrt{\tfrac{1}{4}q^2 + \tfrac{1}{27}p^3}} + \sqrt[3]{-\tfrac{1}{2}q - \sqrt{\tfrac{1}{4}q^2 + \tfrac{1}{27}p^3}}$ is a root of that equation. He pays much attention to the case where $\tfrac{1}{4}q^2 + \tfrac{1}{27}p^3 < 0$

In that case, he points out that the square root $\sqrt{\tfrac{1}{4}q^2 + \tfrac{1}{27}p^3}$ does not exist because square roots of negative numbers do not exist. De Gelder does not use the modern terms real or complex. Instead of i he writes the square root of minus one, $\sqrt{-1}$ On the one hand, the square roots of negative numbers do not exist, but on the other hand, the solution of the cubic equation does exist. He calls this situation a severe difficulty.

De Gelder did not present a numerical example, but in the classroom, he might have given an equation such as $x^3 - 6x + 4 = 0$ to specify the difficulty. It is quite obvious that $x = 2$ is a solution. According to the formula, the solution is $x = \sqrt[3]{-2 + \sqrt{-4}} + \sqrt[3]{-2 - \sqrt{-4}}$ and it is not obvious at all that this is the same as $x = 2$. At first sight, this sum of roots looks like a negative number in combination with a complex number. I give three more examples to underline the importance of the invention of complex numbers. Understanding the roots of complex numbers, one can easily prove that $\sqrt[3]{-2 + \sqrt{-4}}$ is the same as $1 + \sqrt{-1}$ and that $\sqrt[3]{-2 - \sqrt{-4}}$ is the same as $1 - \sqrt{-1}$. This example may give the impression that it is always possible to rewrite the Cardano solution. However, there is no general formula which for given a and b, and for unknown c and d provides the simplification of the expression $\sqrt[3]{-a - b\sqrt{-1}}$ to $c + d\sqrt{-1}$ in which c and d are stated in a and b. De Gelder would have stimulated his readers to solve the equation $\sqrt[3]{-2 + \sqrt{-4}} = c + d\sqrt{-1}$ by means of algebra. Do you think you will find $c = 1$ and $d = 1$? One may also experience De Gelder's severe difficulty when trying to solve the equation $x^3 - 6x + 2 = 0$ and simplifying its root $x = \sqrt[3]{-1 + \sqrt{-7}} + \sqrt[3]{-1 - \sqrt{-7}}$. In this case, there is no integer solution. For the general equation $x^3 + px + q = 0$, De Gelder investigates the possible simplification of a root $x = \sqrt[3]{h + k\sqrt{-1}} + \sqrt[3]{h - k\sqrt{-1}}$ where $h = -\frac{1}{2}q$ and $-k^2 = \frac{1}{4}q^2 + \frac{1}{27}p^3$. He tries to do this by raising this cubic root to the third power.

Here are the successive steps in his reasoning.

$$x=\sqrt[3]{h+k\sqrt{-1}}+\sqrt[3]{h-k\sqrt{-1}}$$
$$x^3=\left(\sqrt[3]{h+k\sqrt{-1}}+\sqrt[3]{h-k\sqrt{-1}}\right)^3$$
$$x^3=\left(\sqrt[3]{h+k\sqrt{-1}}\right)^3+3\left(\sqrt[3]{h+k\sqrt{-1}}\right)^2\cdot\left(\sqrt[3]{h-k\sqrt{-1}}\right)+3\left(\sqrt[3]{h+k\sqrt{-1}}\right)\cdot\left(\sqrt[3]{h-k\sqrt{-1}}\right)^2+\left(\sqrt[3]{h-k\sqrt{-1}}\right)^3$$
$$x^3=\left(h+k\sqrt{-1}\right)+3\cdot\sqrt[3]{h+k\sqrt{-1}}\cdot\sqrt[3]{h-k\sqrt{-1}}\cdot\left(\sqrt[3]{h+k\sqrt{-1}}+\sqrt[3]{h-k\sqrt{-1}}\right)+\left(h-k\sqrt{-1}\right)$$
$$x^3=2h+3\cdot\sqrt[3]{\left(h+k\sqrt{-1}\right)\left(h-k\sqrt{-1}\right)}\cdot\left(\sqrt[3]{h+k\sqrt{-1}}+\sqrt[3]{h-k\sqrt{-1}}\right)$$
$$x^3=2h+3\cdot\sqrt[3]{h^2+k^2}\cdot\left(\sqrt[3]{h+k\sqrt{-1}}+\sqrt[3]{h-k\sqrt{-1}}\right)$$
$$x^3=2h+3\cdot\sqrt[3]{h^2+k^2}\cdot x$$
$$x^3-3\cdot\sqrt[3]{h^2+k^2}\cdot x-2h=0$$

In the sixth line, there are still two cube roots. One of them is the unknown x. The final line is therefore a cubic equation, but things are even worse. Because $h=-\frac{1}{2}q$ and $-k^2=\frac{1}{4}q^2+\frac{1}{27}p^3$ the equation $x^3-3\cdot\sqrt[3]{h^2+k^2}\cdot x-2h=0$ can be rewritten as $x^3+px+q=0$. Note, this is the equation De Gelder started with. So, this is the unavoidable loop. Although a real solution exists, which can be very simple in some cases, there is no general way to simplify the formula. It is not possible to get rid of the sum of cube roots and in the case of three real roots, it is not possible to get rid of the complex numbers.

Therefore, De Gelder calls the case $\frac{1}{4}q^2+\frac{1}{27}p^3<0$ the irreducible case. He stresses that irreducible does not mean that you cannot solve the equation. It rather means that you cannot express the result of the formula, the sum of two cube roots as one number or one single cube root.

De Gelder was aware that his readers might despair because it might seem to them that they got stuck in an endless loop of rewriting expressions, always returning to a similar cubic equation with a square root that did not exist.

When we elaborate on the example of the equation $x^3-6x+4=0$ we will get the same unfortunate result. We start with the Cardano solution, raise it to the third power and rewrite the expression. So $p=-6$ and $q=4$ therefore $h=-2$ and $k=-4$. In the fifth line, at the right, we recognize our x, the Cardano solution. So we end up with the original equation and there is no trace of the simple solution

$x = 2$. See the following deduction and check that the final line is identical to the equation we started with.

$$x = \sqrt[3]{-2+\sqrt{-4}} + \sqrt[3]{-2-\sqrt{-4}}$$

$$x^3 = \left(\sqrt[3]{-2+\sqrt{-4}} + \sqrt[3]{-2-\sqrt{-4}}\right)^3$$

$$x^3 = \left(\sqrt[3]{-2+\sqrt{-4}}\right)^3 + 3\left(\sqrt[3]{-2+\sqrt{-4}}\right)^2 \cdot \left(\sqrt[3]{-2-\sqrt{-4}}\right) + 3\left(\sqrt[3]{-2+\sqrt{-4}}\right) \cdot \left(\sqrt[3]{-2-\sqrt{-4}}\right)^2 + \left(\sqrt[3]{-2-\sqrt{-4}}\right)^3$$

$$x^3 = \left(-2+\sqrt{-4}\right) + 3\sqrt[3]{-2+\sqrt{-4}} \cdot \sqrt[3]{-2-\sqrt{-4}} \cdot \left(\sqrt[3]{-2+\sqrt{-4}} + \sqrt[3]{-2-\sqrt{-4}}\right) + \left(-2-\sqrt{-4}\right)$$

$$x^3 = -4 + 3\sqrt[3]{\left(-2+\sqrt{-4}\right)\left(-2-\sqrt{-4}\right)} \cdot \left(\sqrt[3]{-2+\sqrt{-4}} + \sqrt[3]{-2-\sqrt{-4}}\right)$$

$$x^3 = -4 + 3\sqrt[3]{8} \cdot x$$

$$x^3 = -4 + 6x$$

$$x^3 - 6x + 4 = 0$$

De Gelder stimulated his pupils and readers to investigate mathematical truth themselves. He might have given another example, like the equation $x^3 - 6x + 2 = 0$ in order that they experience the severe difficulty, the endless loop and the despair. You may try to find all three real solutions by means of algebra.

After this algebraic approach, De Gelder continued with numerical approximations and with a trigonometric solution in order to present a complete mathematical overview of cubic equations. It is almost unbelievable that young pupils aged 13 - 15 years were able to master this kind of mathematics.

Influence

After De Gelder, other Dutch authors, like Lobatto (1797–1866), Van der Ven (1833–1909), Rahusen (1860–1917), Schuh (1875–1966) and Wijdenes (1872–1972) followed a similar mathematical approach, but words such as "zwarigheid" and "wanhopen" were not used, except by Van der Ven (Ven, 1864). These textbooks contained short monologues instead of Socratic reasoning. Every next generation of textbooks paid less attention to the step from Cardano's single real solution to the three cube roots and the problem of the endless loop. Van der Ven stated in his preface that he wrote his book *De theorie en de oplossing van hoogere magtsvergelijkingen* [Theory and solution of higher degree equations] (Ven, 1864) after he had heard many complaints about the excessive shortness of other authors when dealing with higher order equations.

De Gelder wrote in the preface that his books were intended for young pupils and their teachers. The books mentioned above are mostly addressed to an older audience, like students, teachers and those who wanted to become a teacher.

In the twentieth century, the cubic equation was no longer considered to be of much interest. After the fifties, the curriculum changed many times, but the interest in the algebraic solution of third-degree polynomial equations did not return. Nowadays, in the Netherlands, pupils use graphical calculators to solve cubic equations numerically. The story of Cardano and Tartaglia might be told by teachers who take an interest in the history of mathematics, but there is no time to explore the topic as extensively as De Gelder did. Nevertheless, mathematical reasoning is still part of the Dutch secondary school curriculum, as De Gelder would have appreciated.

Discussion

In my opinion, De Gelder's pedagogical ideas are still valid and his approach of solving cubic equations is of interest for modern mathematics teaching, because of the combination of its mathematical richness and its didactical approach. It would be worthwhile to investigate how teachers, students, and pupils would gain more insight into mathematical reasoning and problem solving when they attend a course based on De Gelder's ideas and approach. Within the ordinary Dutch secondary school curriculum, there is not much room for such a lesson series, but there is room in the extended math curriculum (labeled math D in the Netherlands). Pupils attending this program do have an interest in a broader and deeper mathematical knowledge. If I ever will teach such a group of pupils about complex numbers, I would like to develop a lesson series about the difficulties of solving cubic equations through the centuries. For mathematics students and those who want to become a teacher in mathematics, this topic would be of interest too.

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An overview of publications of Jacob de Gelder is on my website: see http://www.fransvanschooten.nl/Gelder.htm

1 A transcription of *Verhandeling* is available at www.jphogendijk.nl/sources/gelder.html