a Tradition of Curve Tracing

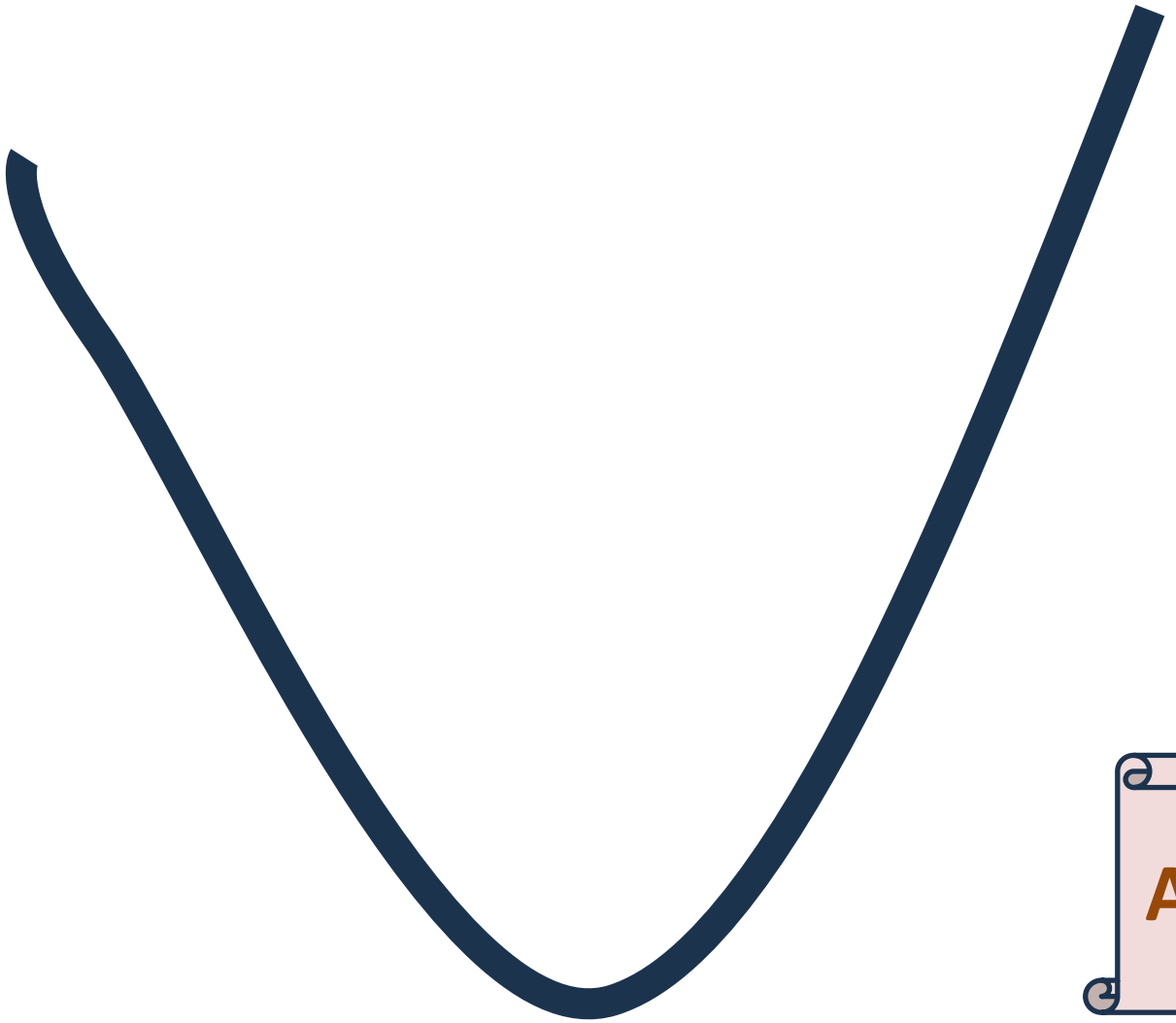
Henk Hietbrink

independent researcher

teacher of mathematics

15:00

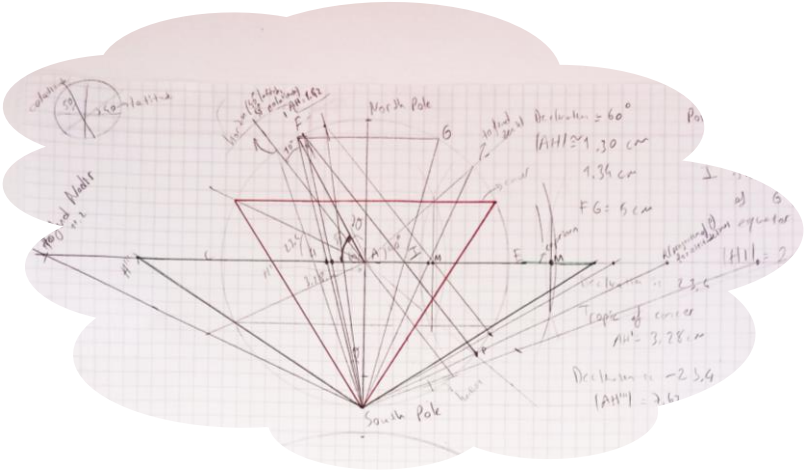
So you think you can draw a perfect parabola in 30 seconds

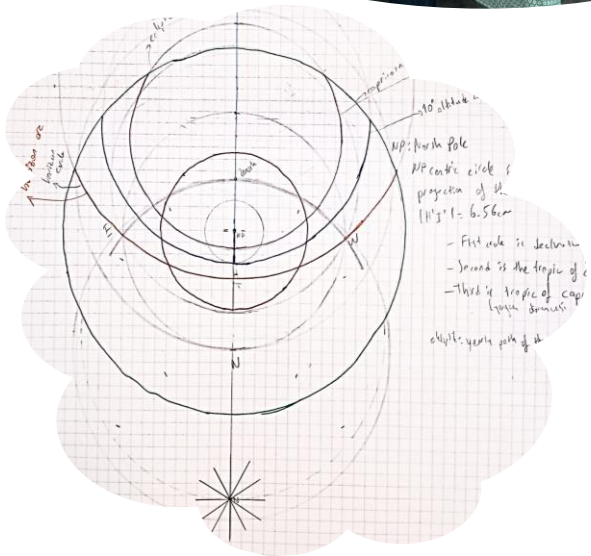


Activity

It started in Istanbul
and finally in

… and finally in Ali Nesin Math Village





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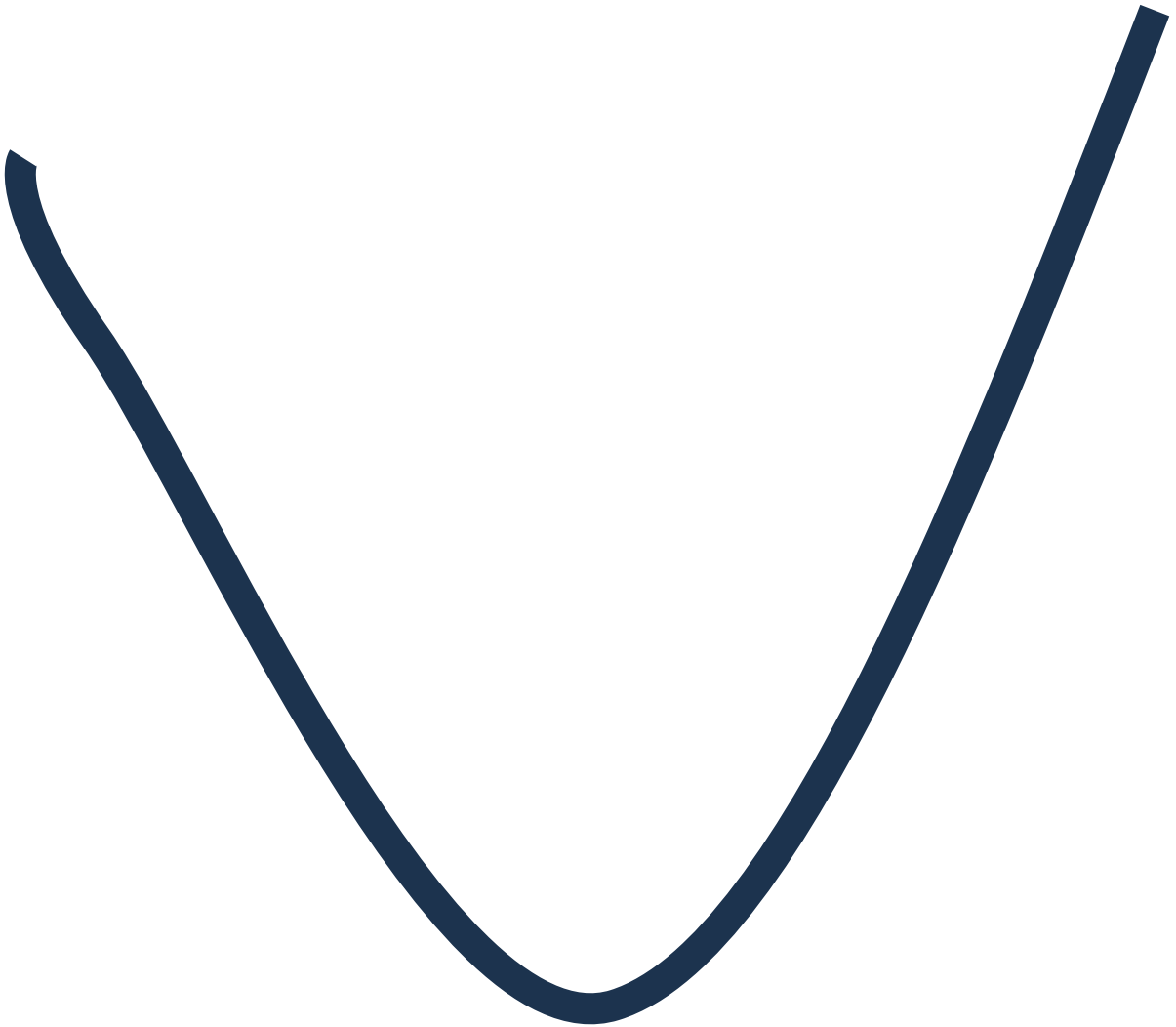

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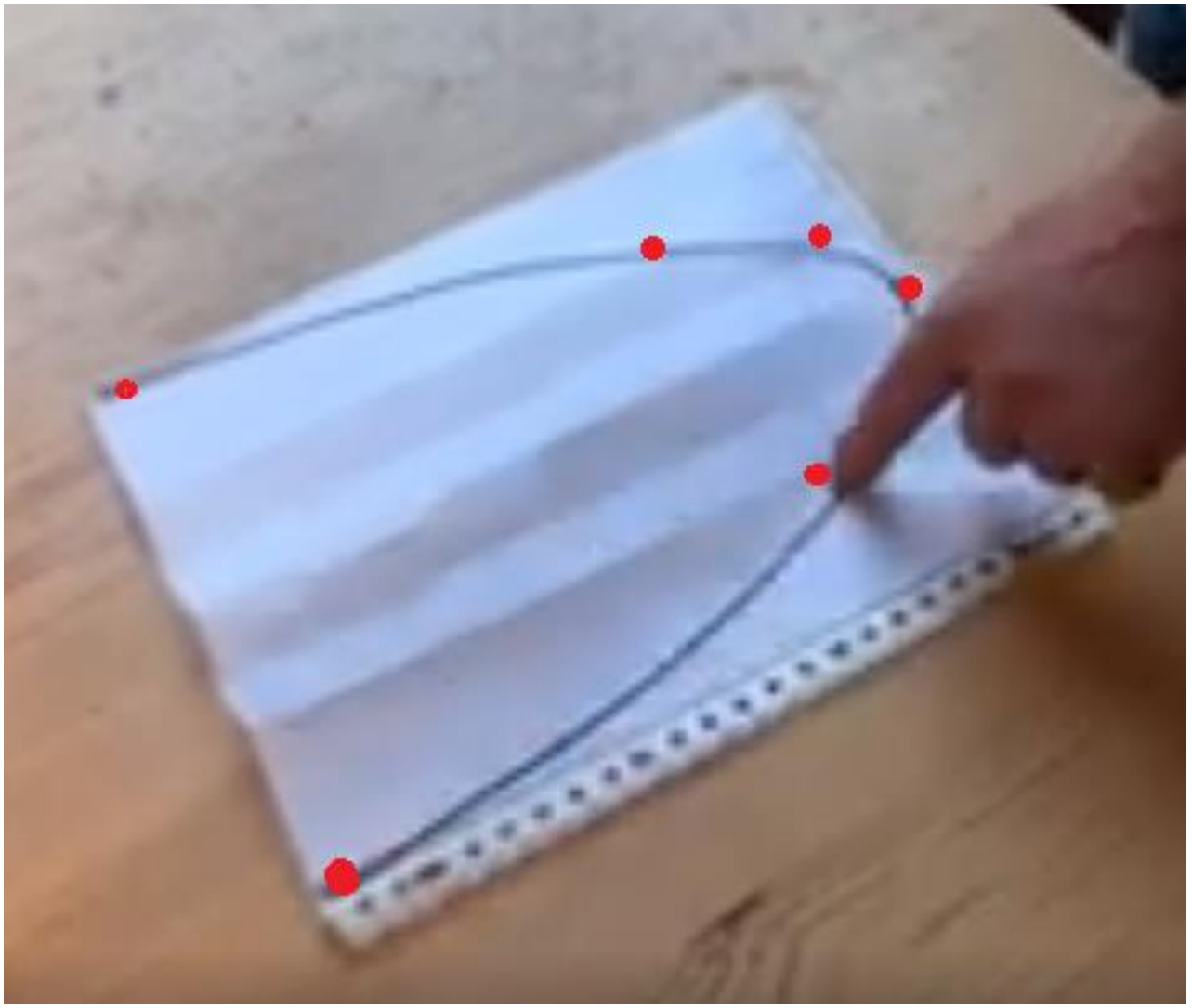
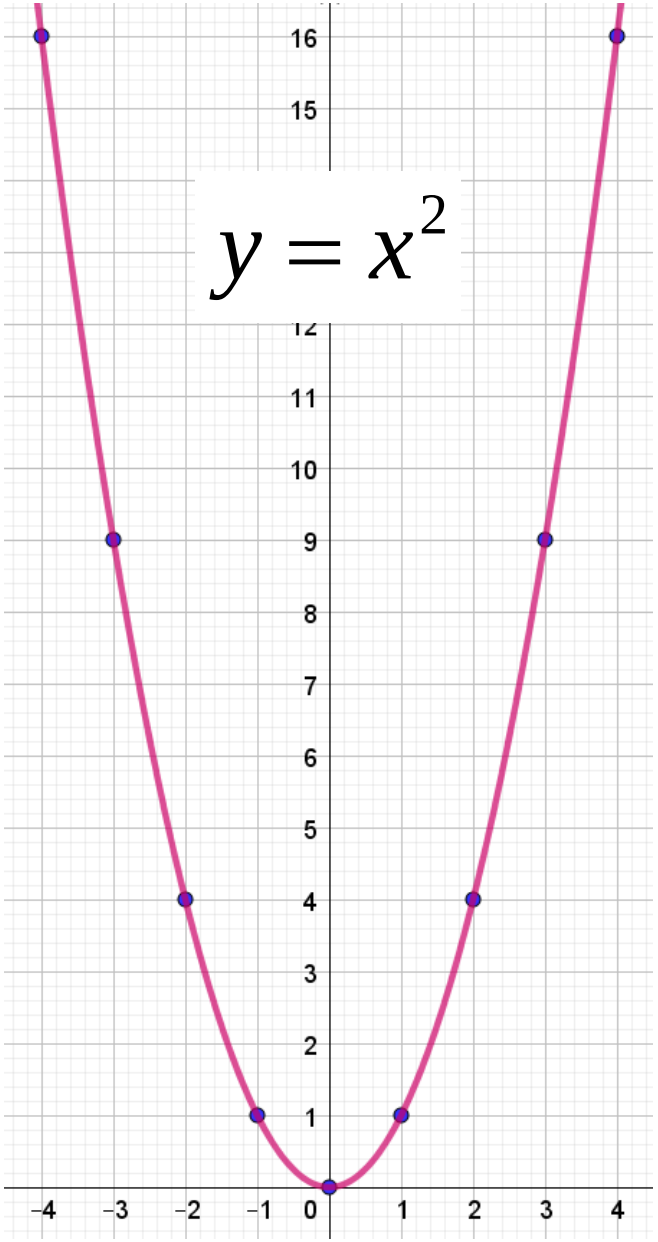
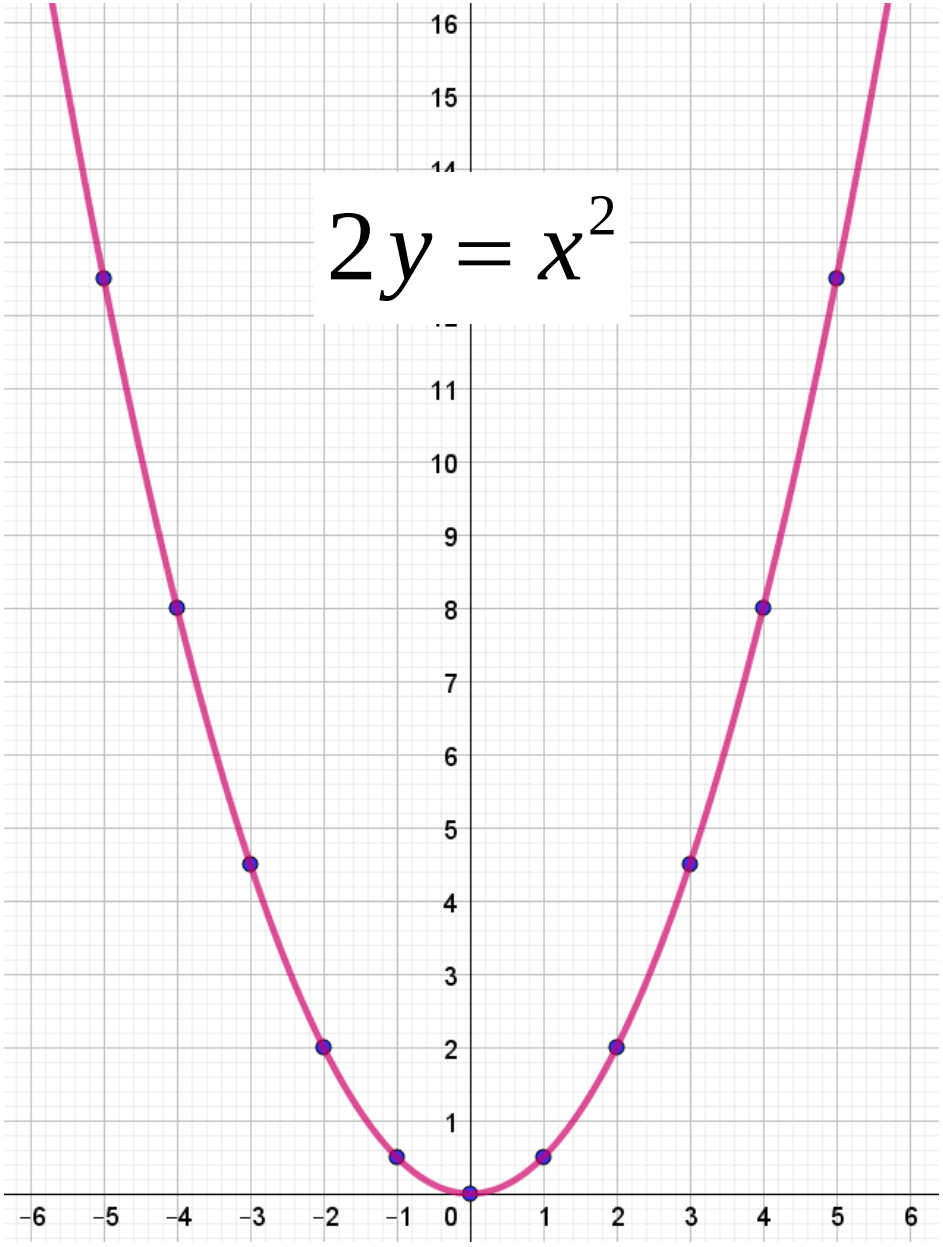


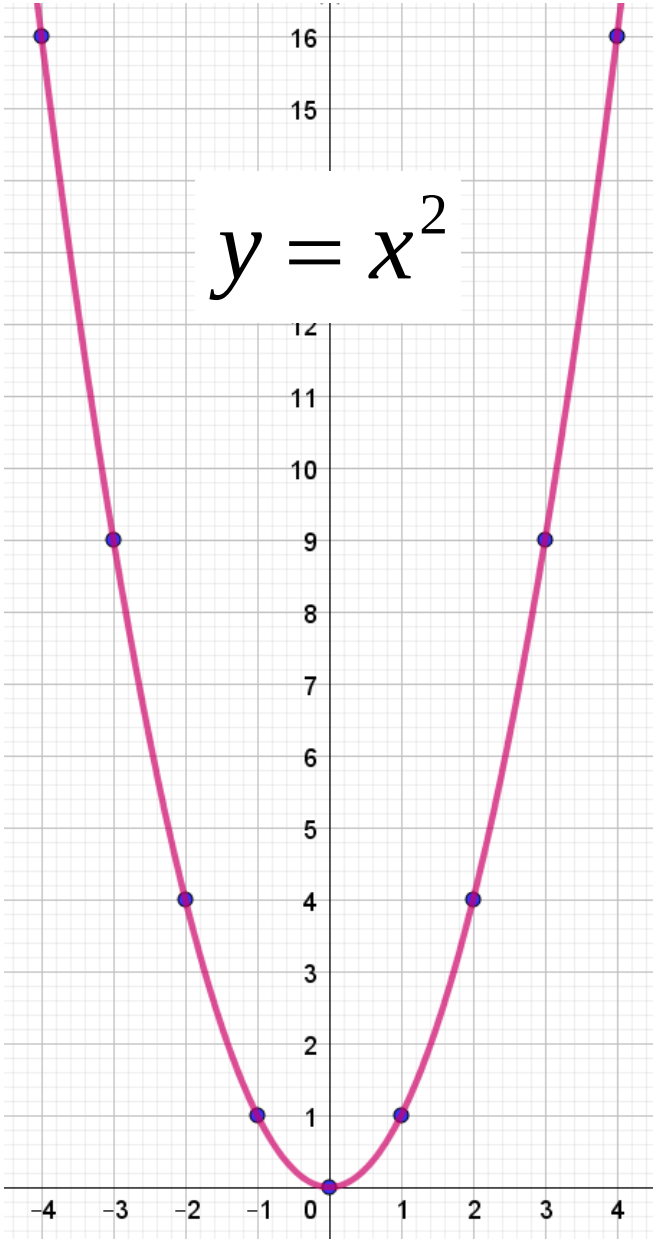
15:05

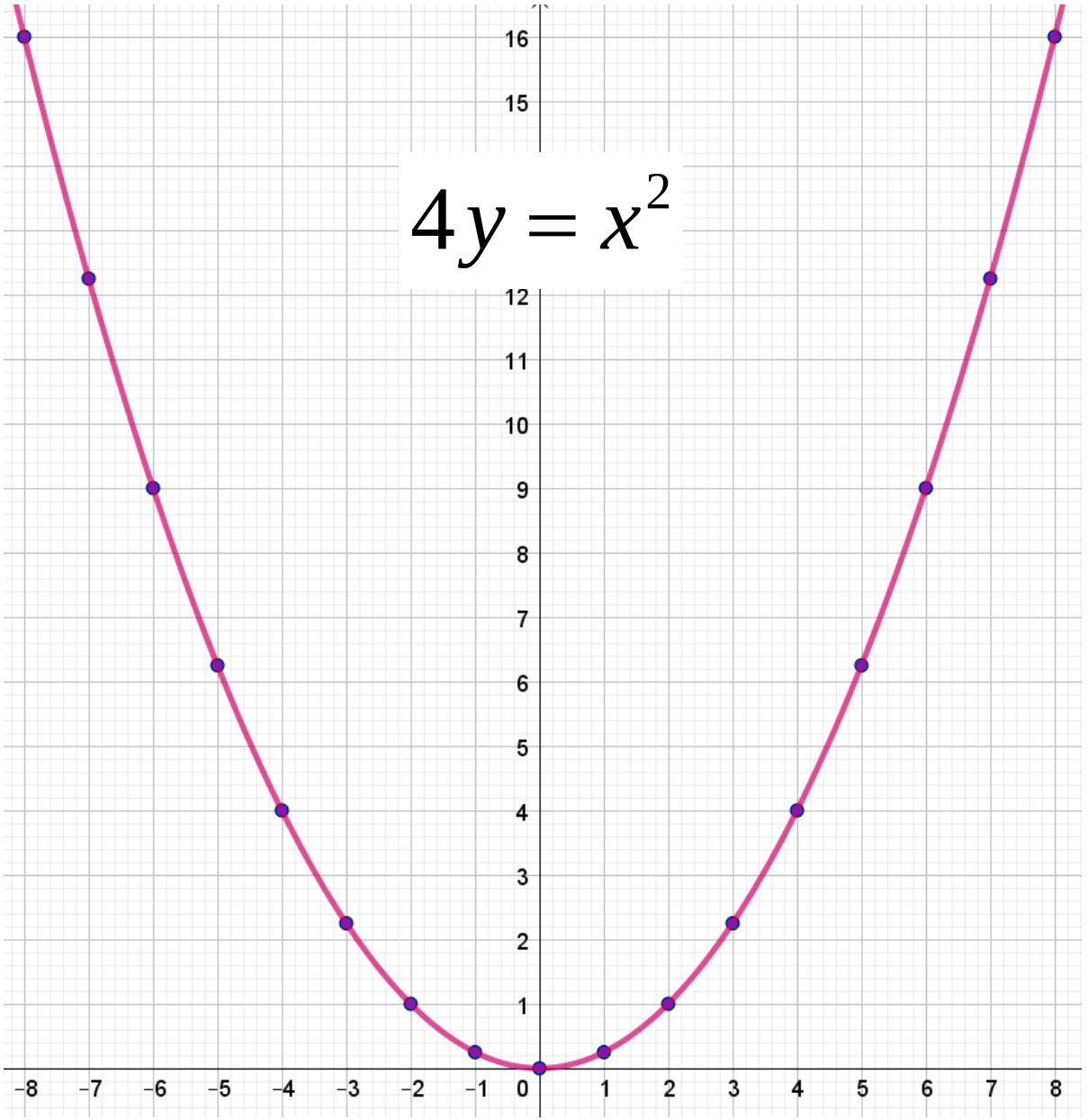
So you think you can draw a perfect parabola in 30 seconds



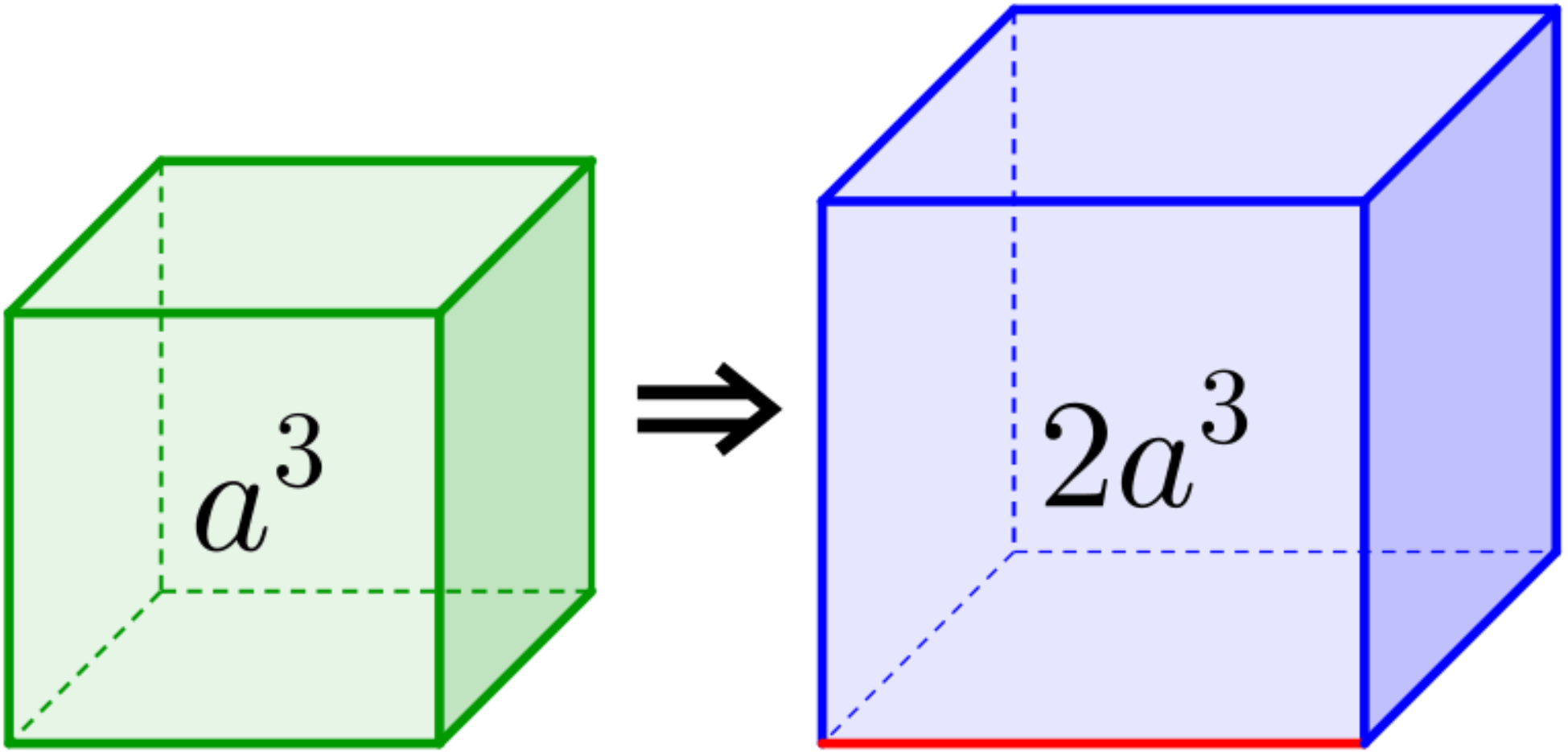


So you think you can draw a perfect parabola in 30 seconds



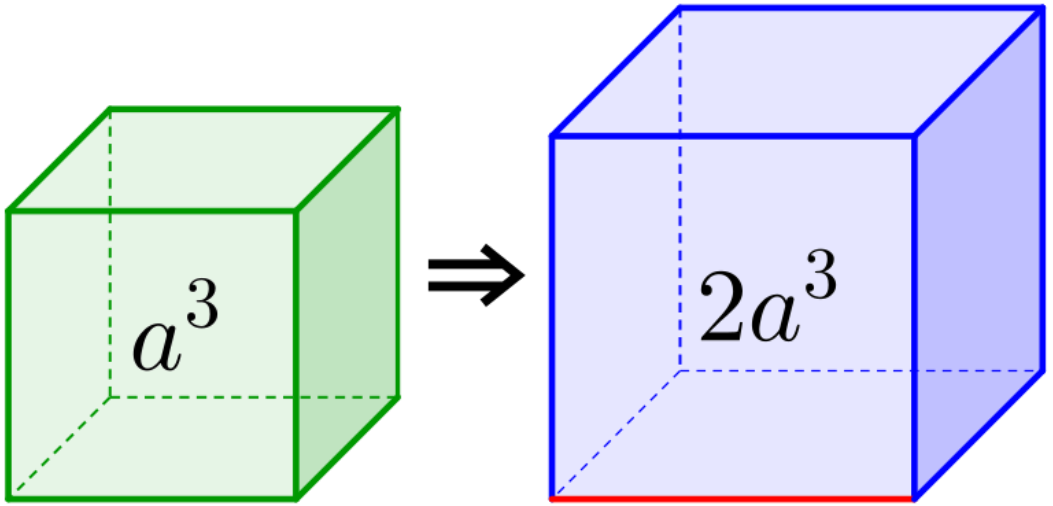


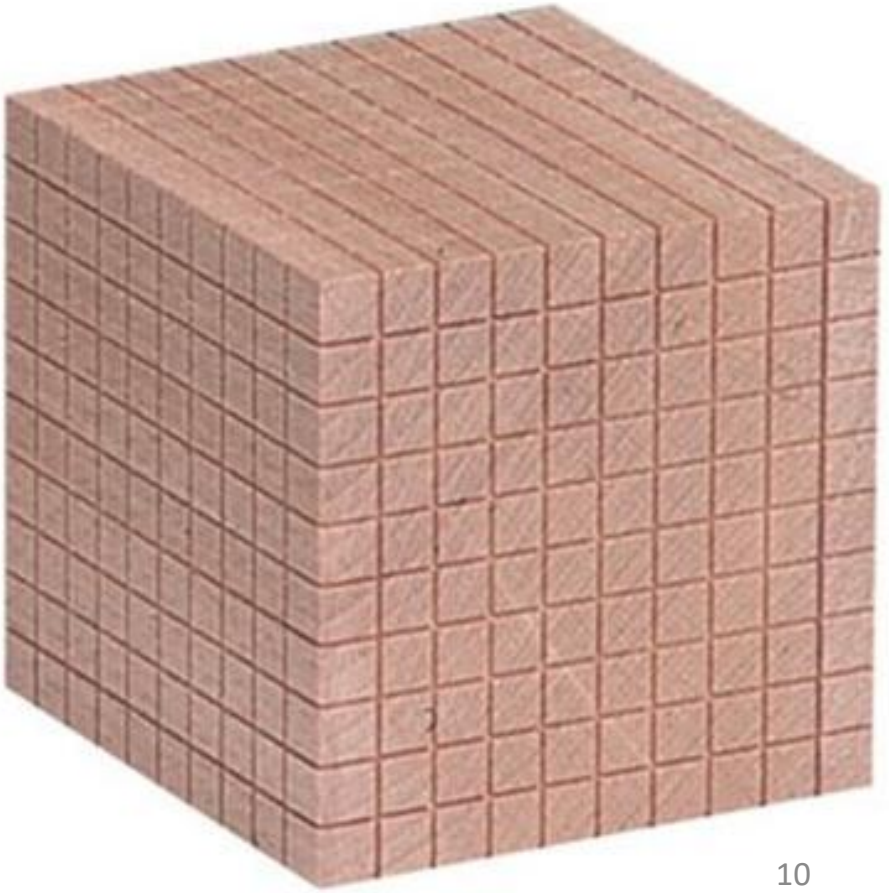
Duplication of the cube



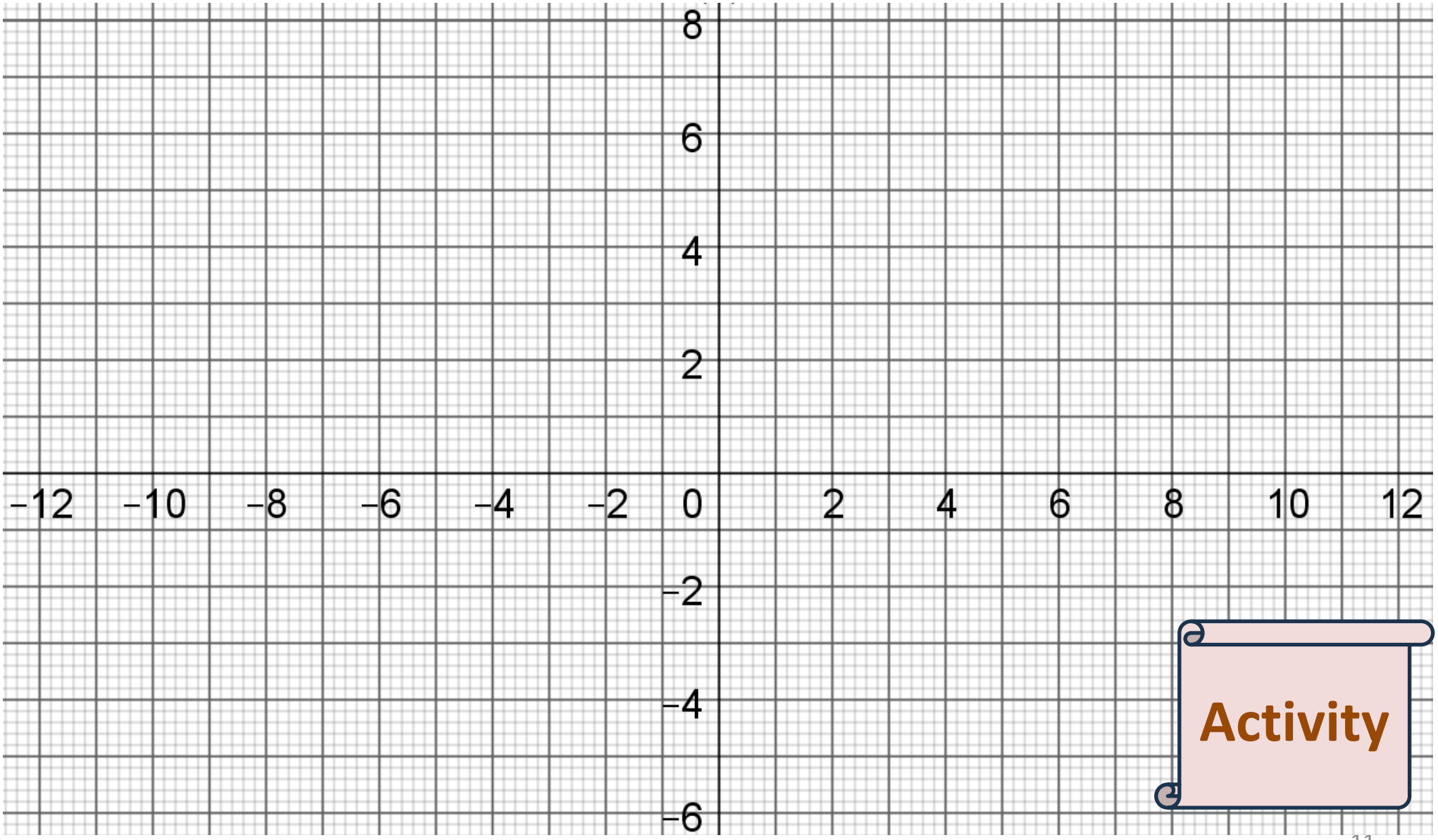
15:10

No integer solution for the duplication of the cube



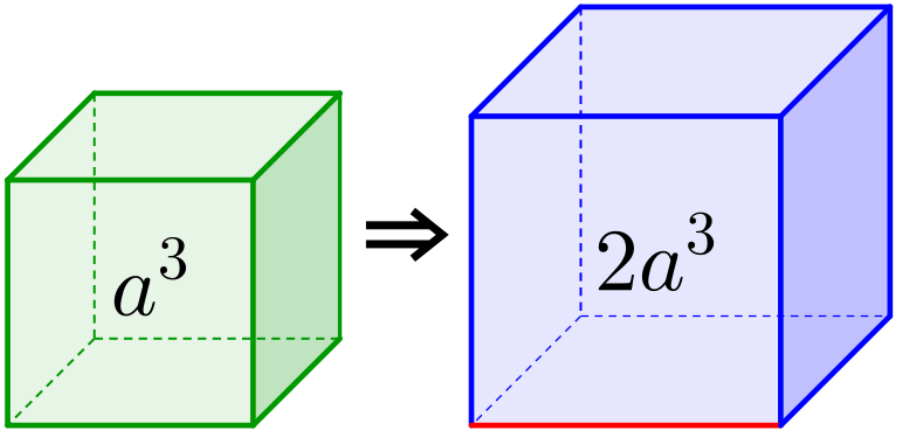


So you think you can solve the cube duplication with a parabola in 30 seconds



Activity

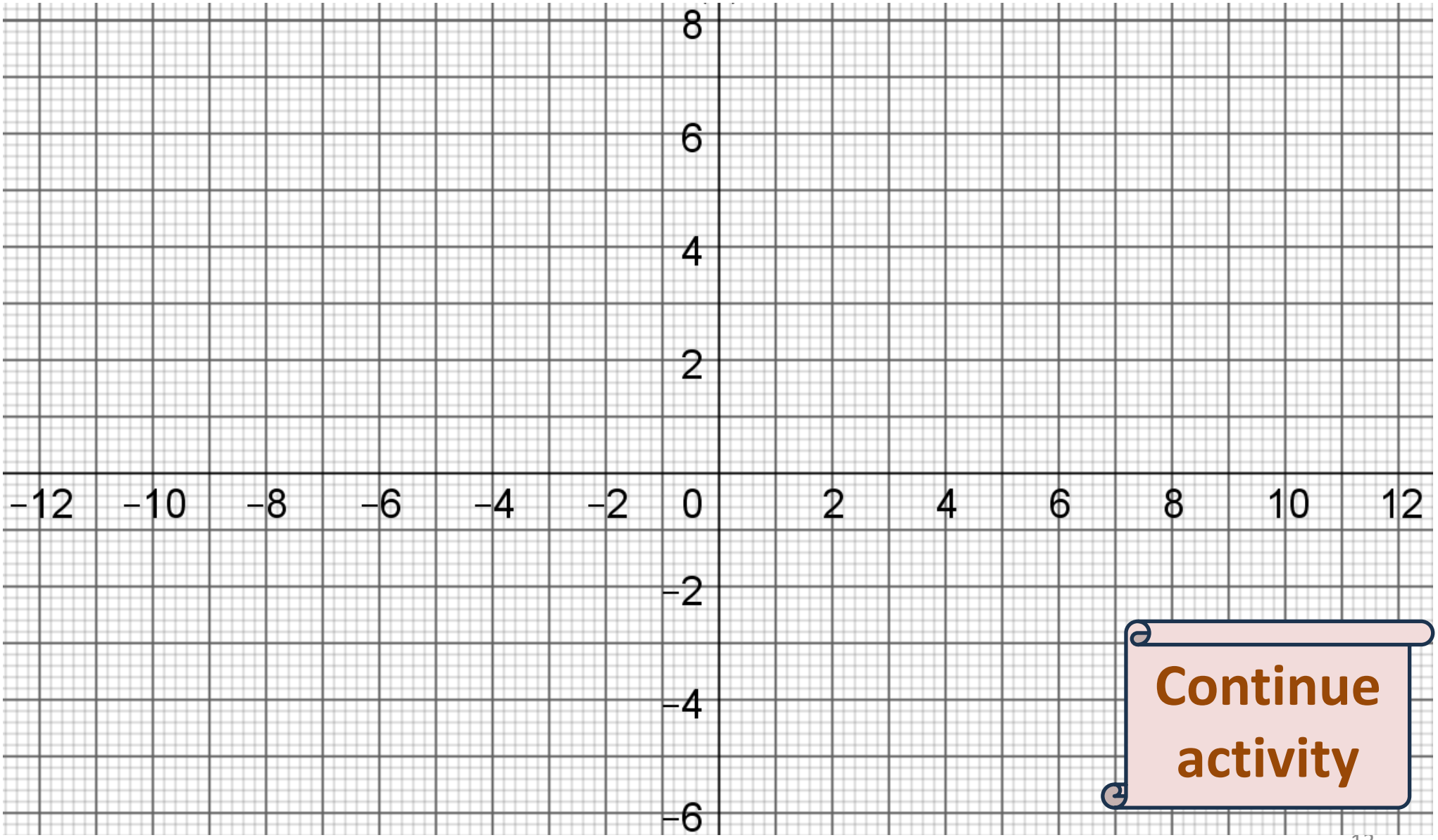
Two parabolas can solve the problem of the duplication of the cube



$$\begin{cases} 2x = y^2 \\ y = x^2 \end{cases} \Rightarrow 2x = (x^2)^2 \Rightarrow 2 = x^3$$

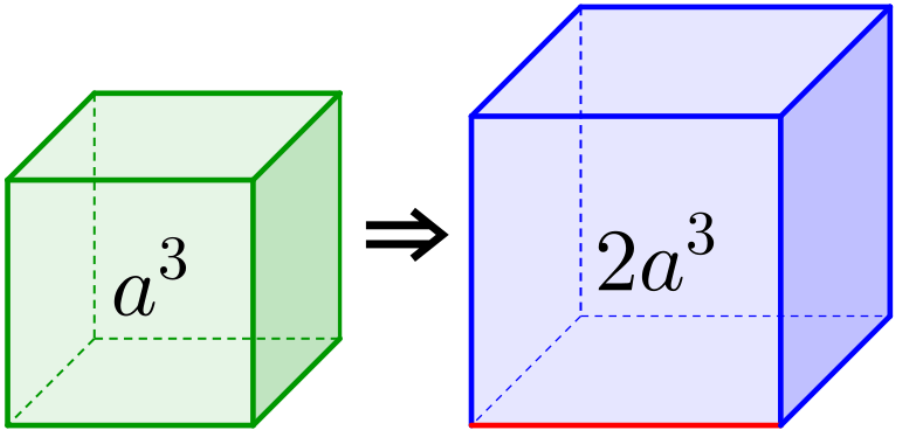
Need help?

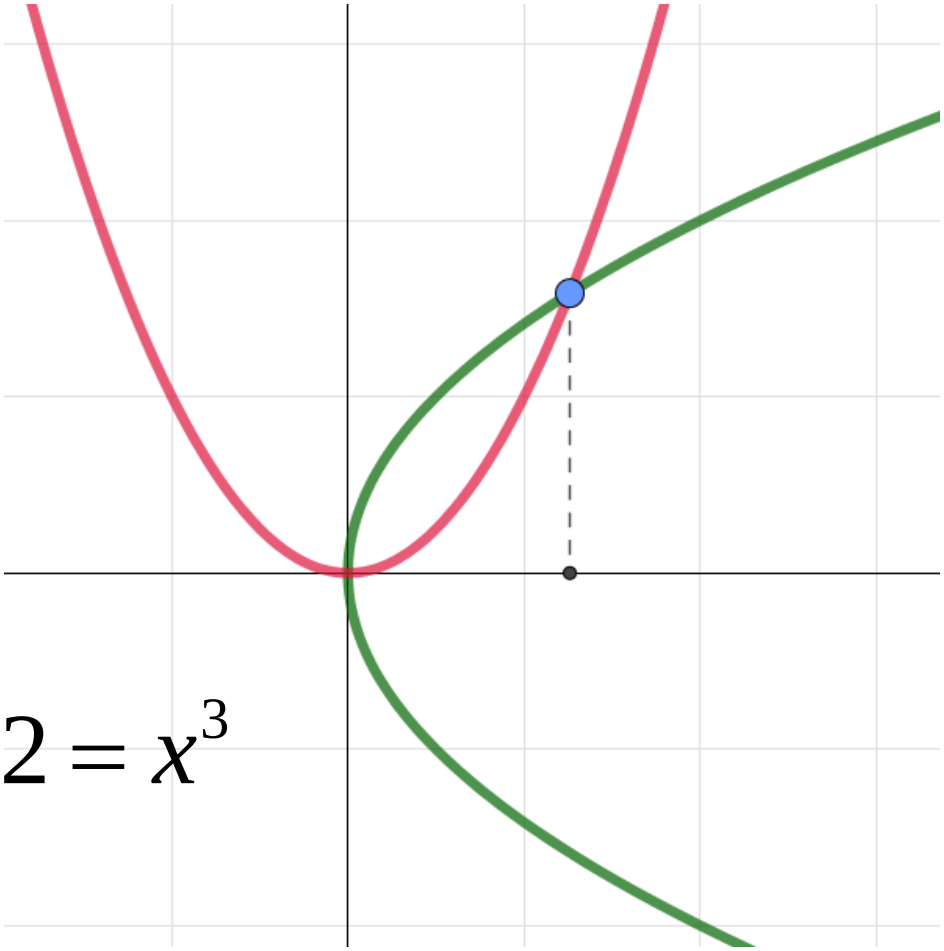
So you think you can solve the cube duplication with a parabola in 30 seconds



Continue activity

Two parabolas can solve the problem of the duplication of the cube

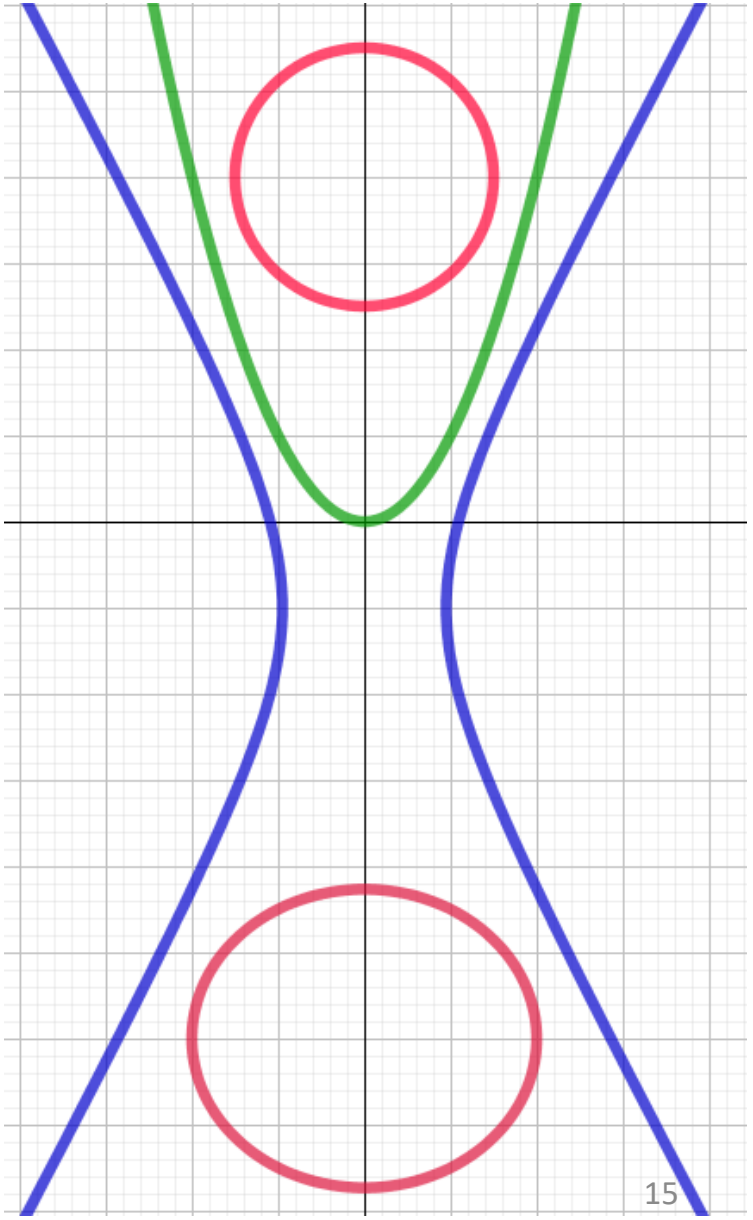




$$\begin{cases} 2x = y^2 \\ y = x^2 \end{cases} \Rightarrow 2x = (x^2)^2 \Rightarrow 2 = x^3$$

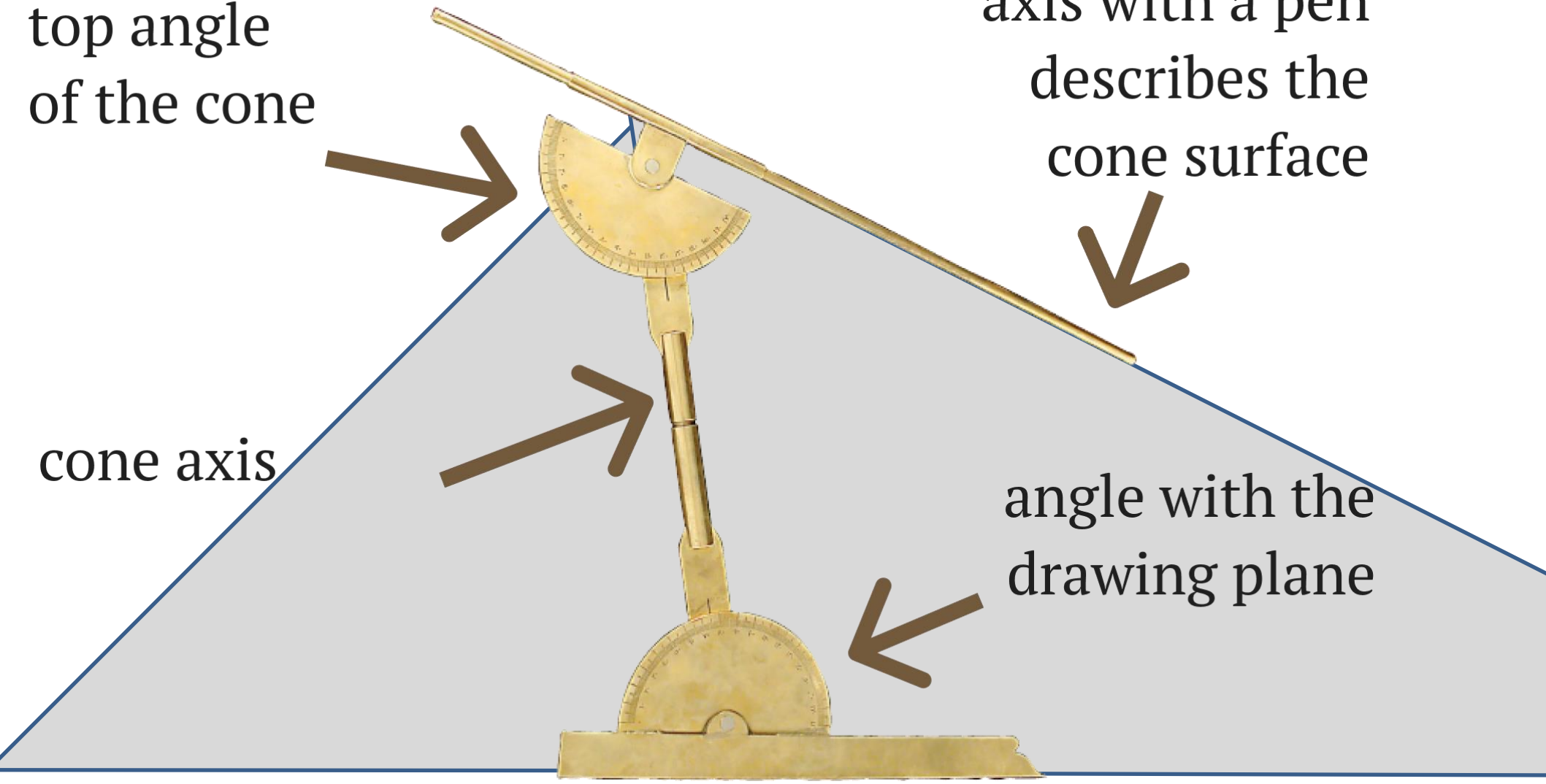
$$\Rightarrow x = \sqrt[3]{2}$$

the Perfect Compass
a device to draw conic sections

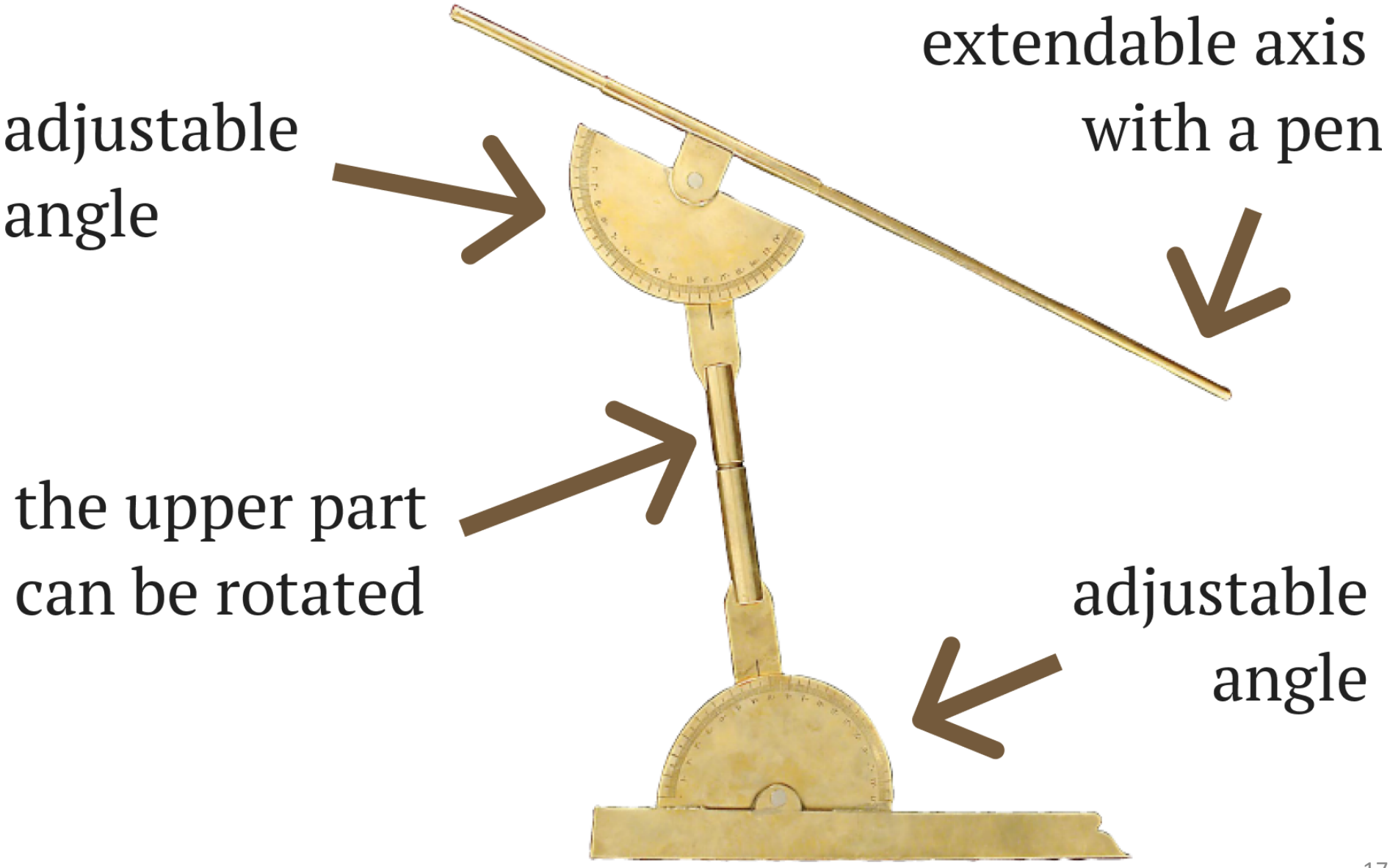


15:20

the Perfect Compass
a device to draw conic sections



the Perfect Compass
a device to draw conic sections

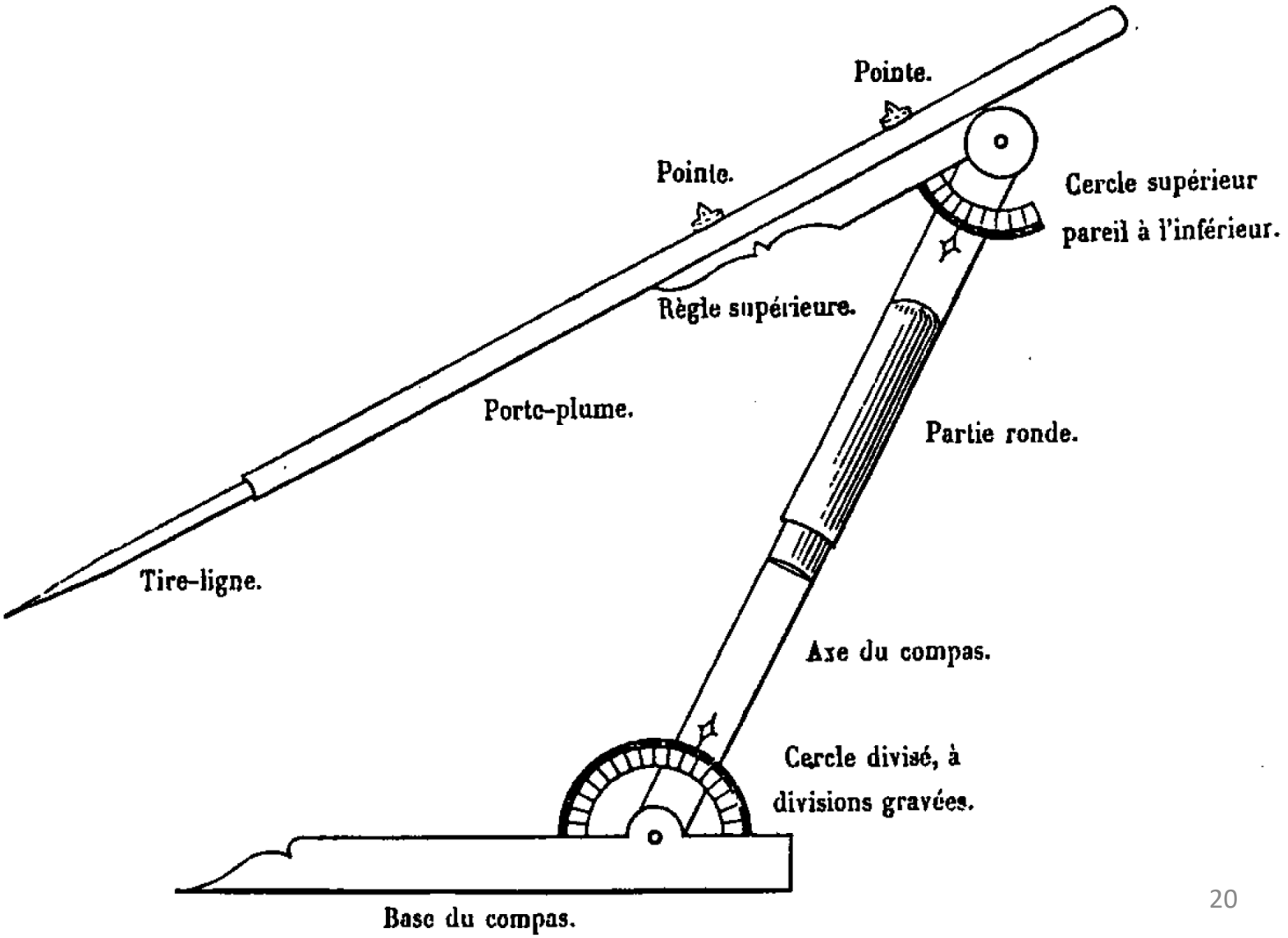


the Perfect Compass
a device to draw conic sections

Doing experiments with a Perfect Compass

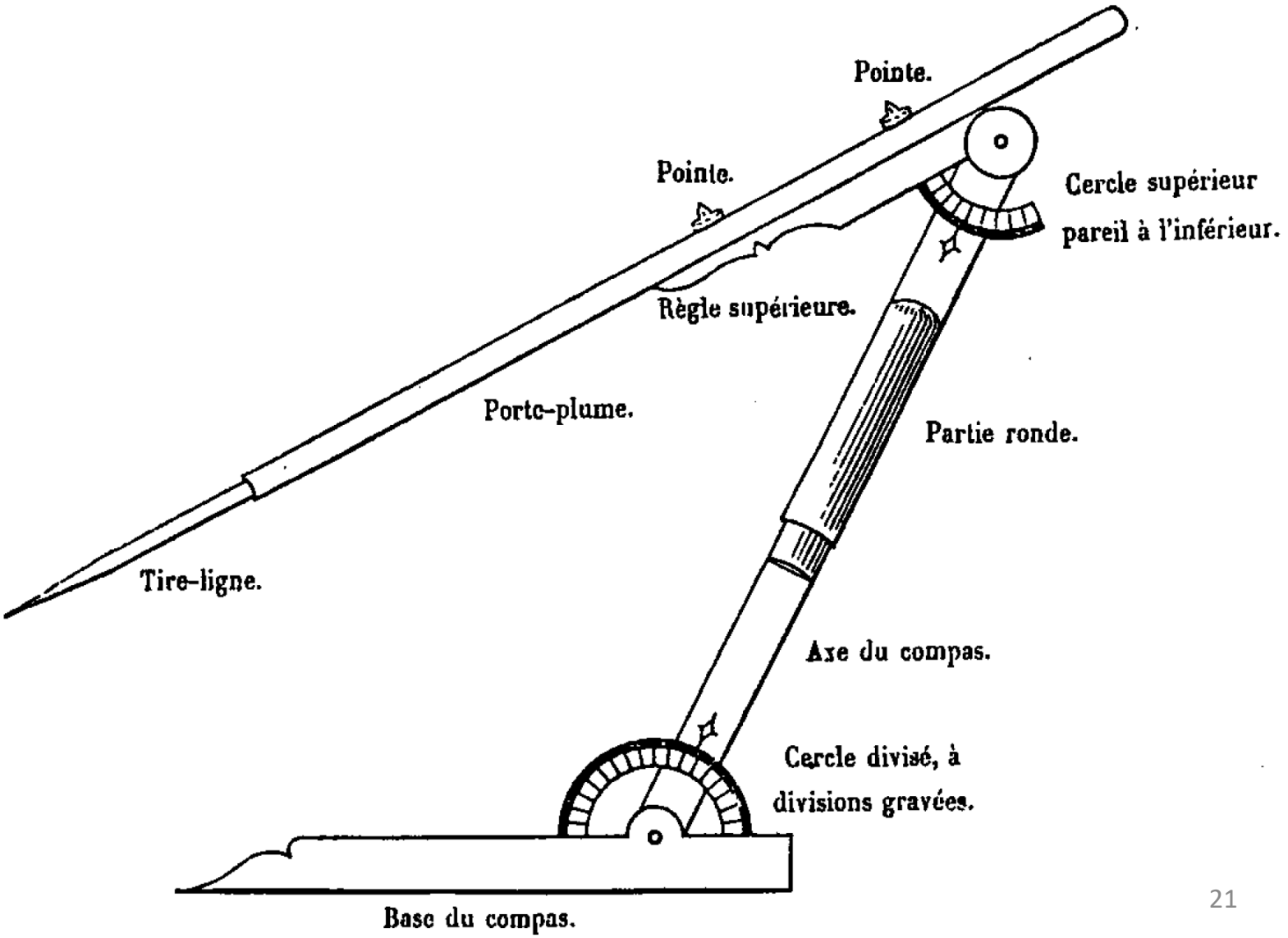
the Perfect Compass
a device to draw conic sections

Fig. 4.



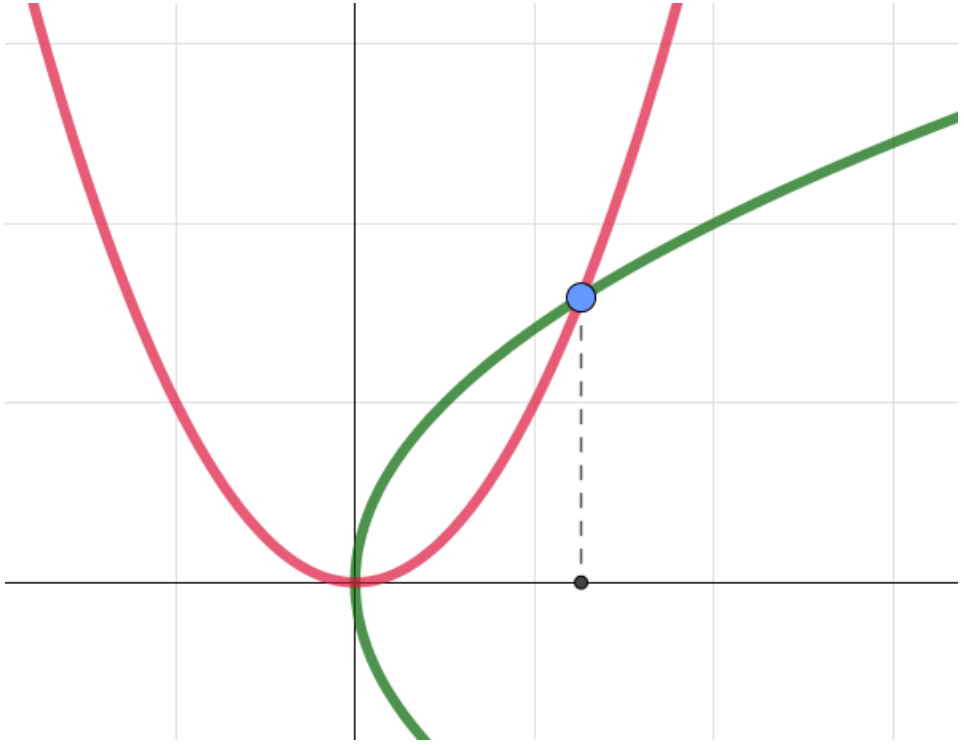
the Perfect Compass
a device to draw conic sections

Fig. 4.



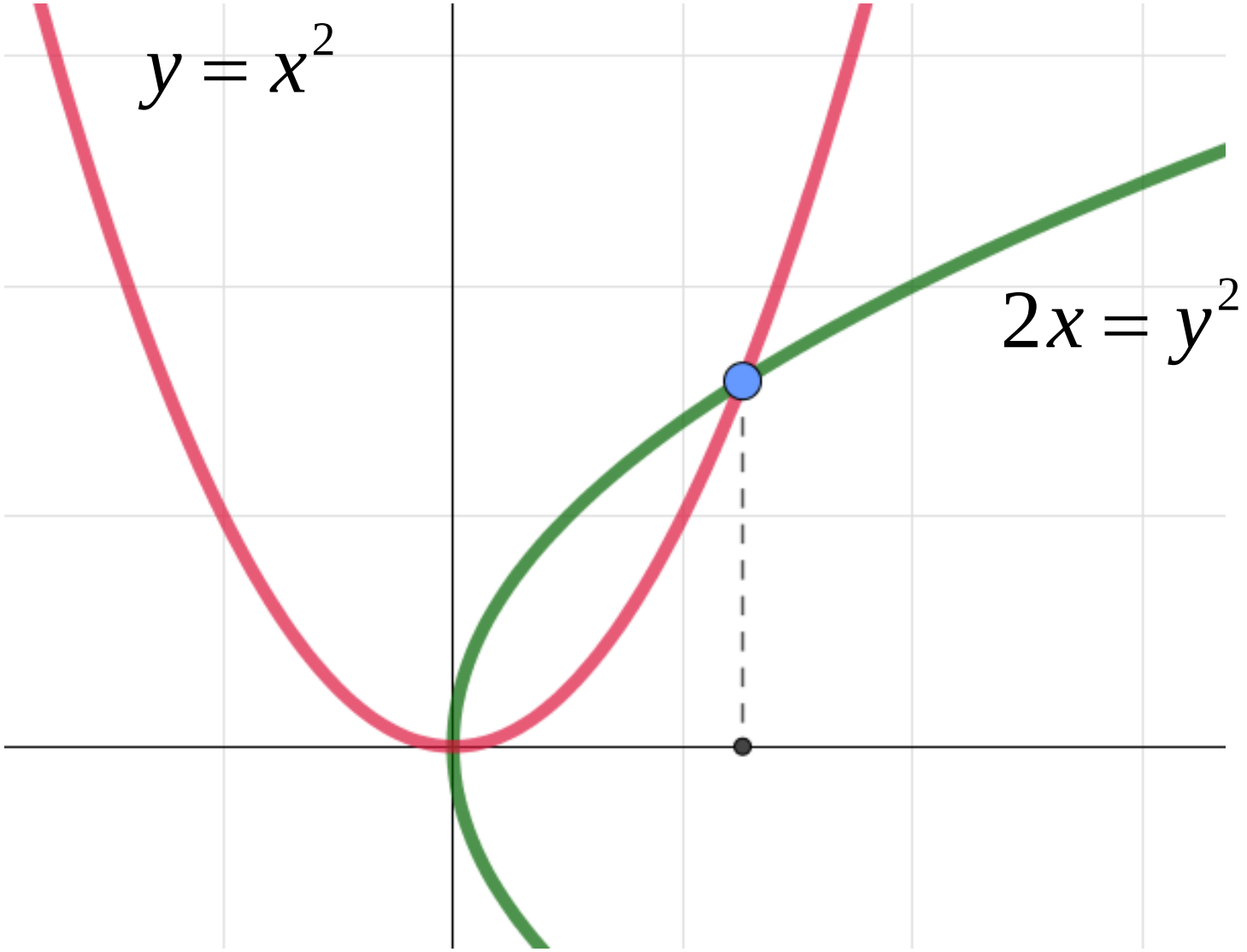
The algebraic solution of the cubic problem Which graph belongs to what curve?

$$\begin{cases} 2x = y^2 \\ y = x^2 \end{cases} \Rightarrow 2x = (x^2)^2 \Rightarrow 2 = x^3 \Rightarrow x = \sqrt[3]{2}$$

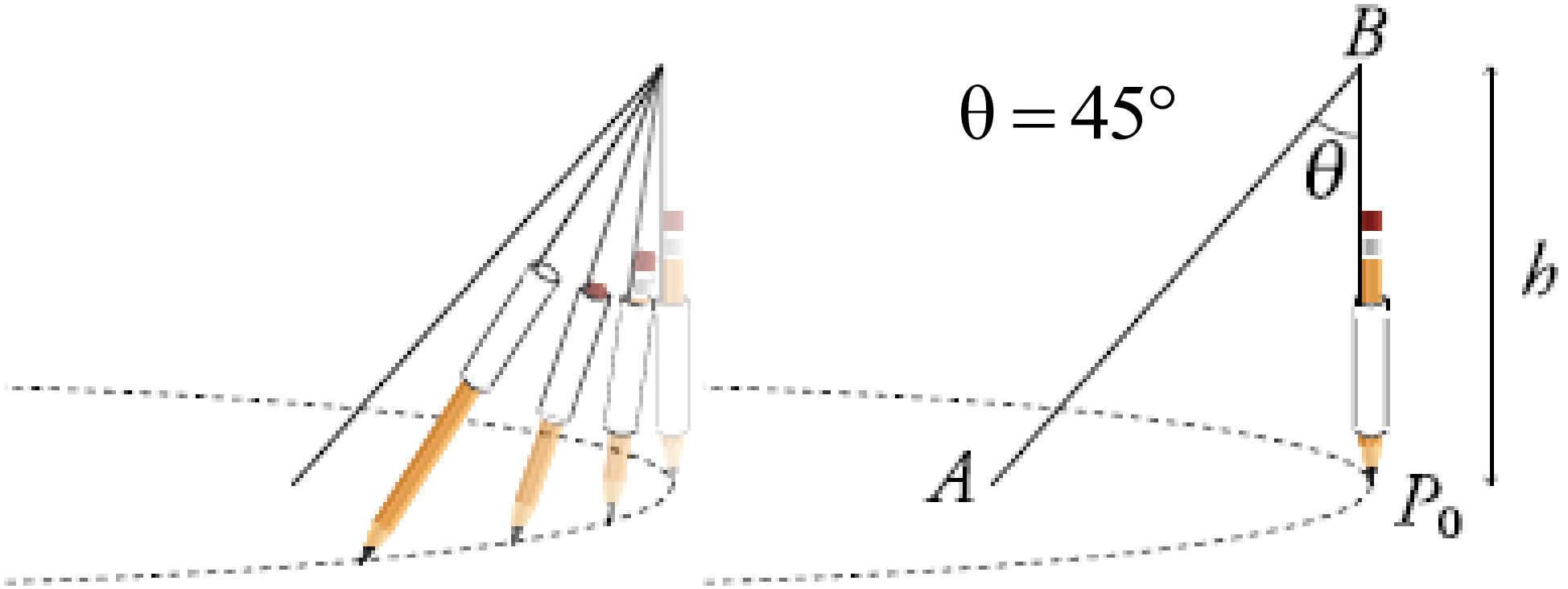


Activity

The algebraic solution of the cubic problem These graphs belongs to parabolas.



Tracing a parabola (© Viktor Blåsjö)



Operationalism: An Interpretation of the Philosophy of Ancient Greek Geometry

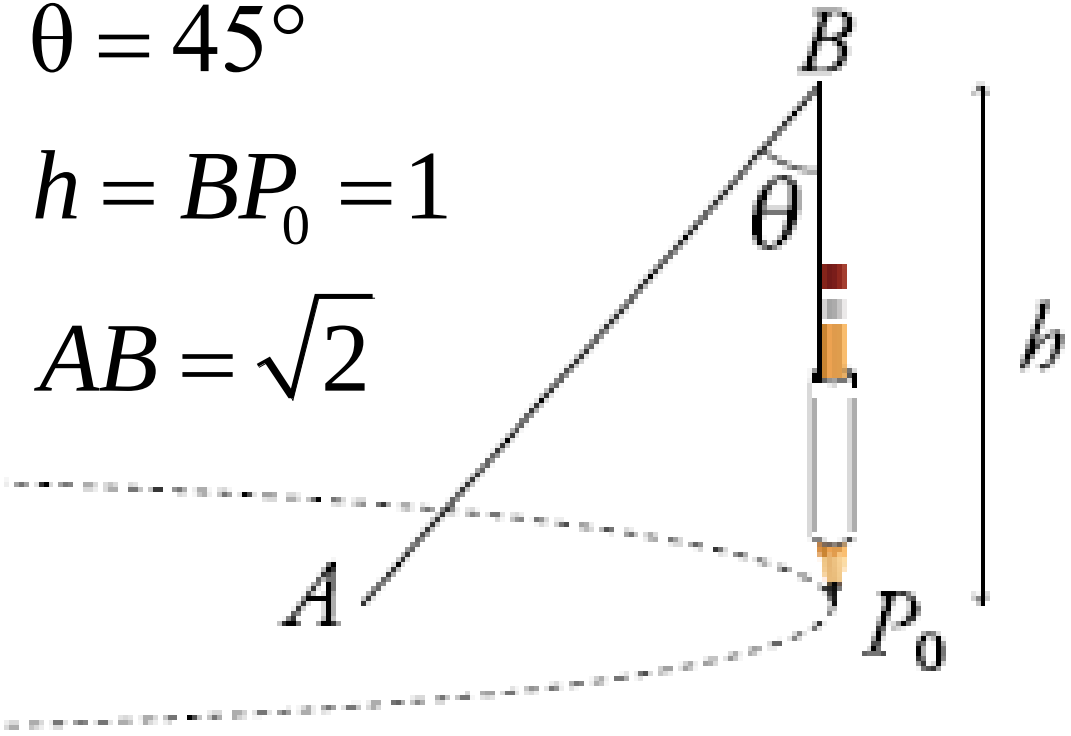
Foundations of Science (2022) 27:587–708
https://doi.org/10.1007/s10699-021-09791-4

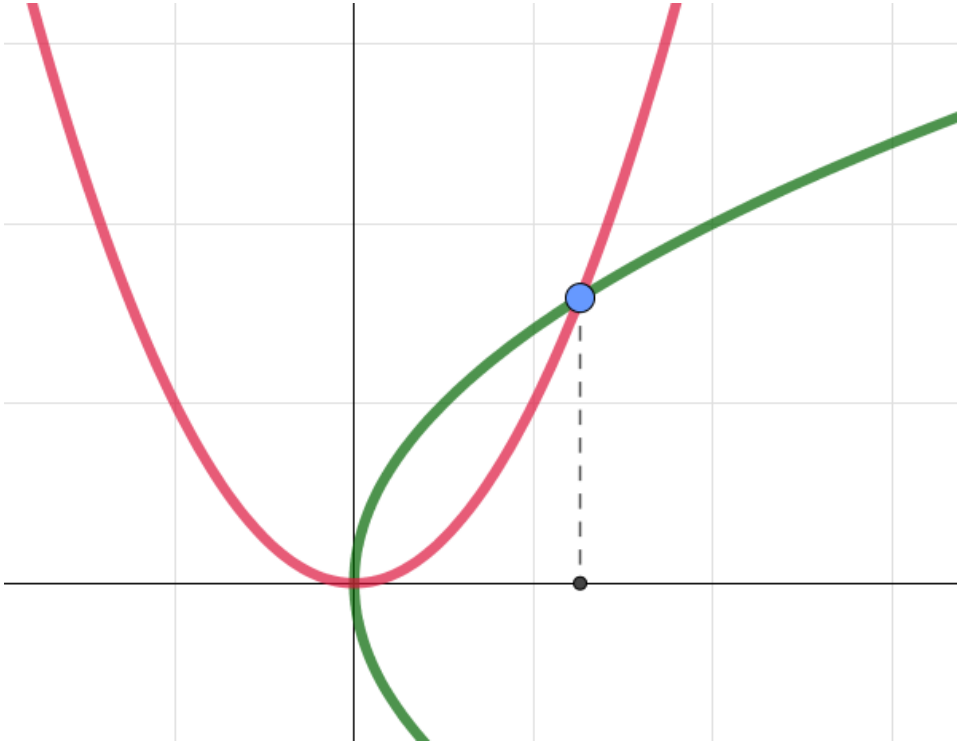
Right angled perfect compass
Which curve can you draw this way?

$\theta = 45°$

$h = BP_0 = 1$

$AB = \sqrt{2}$



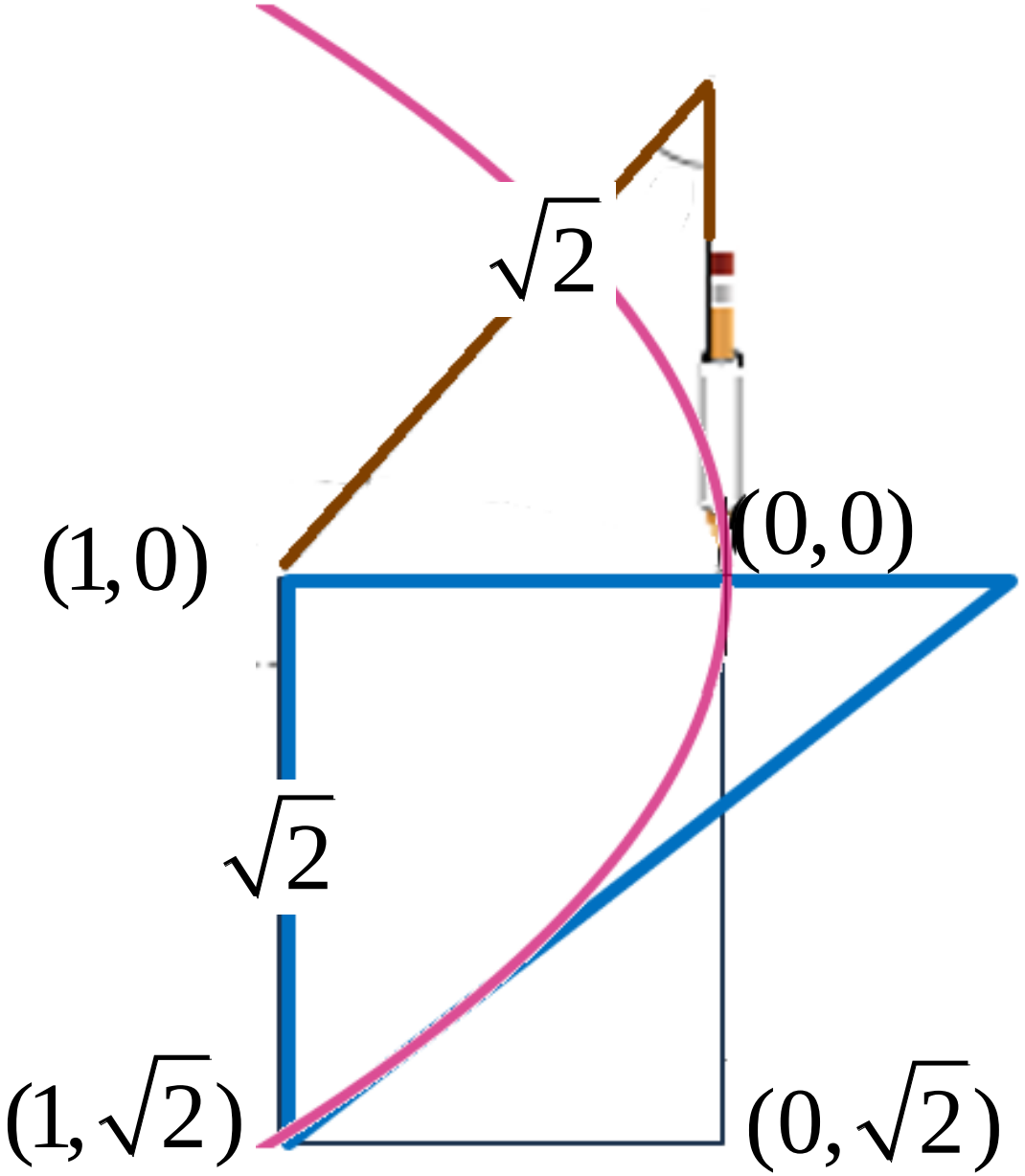


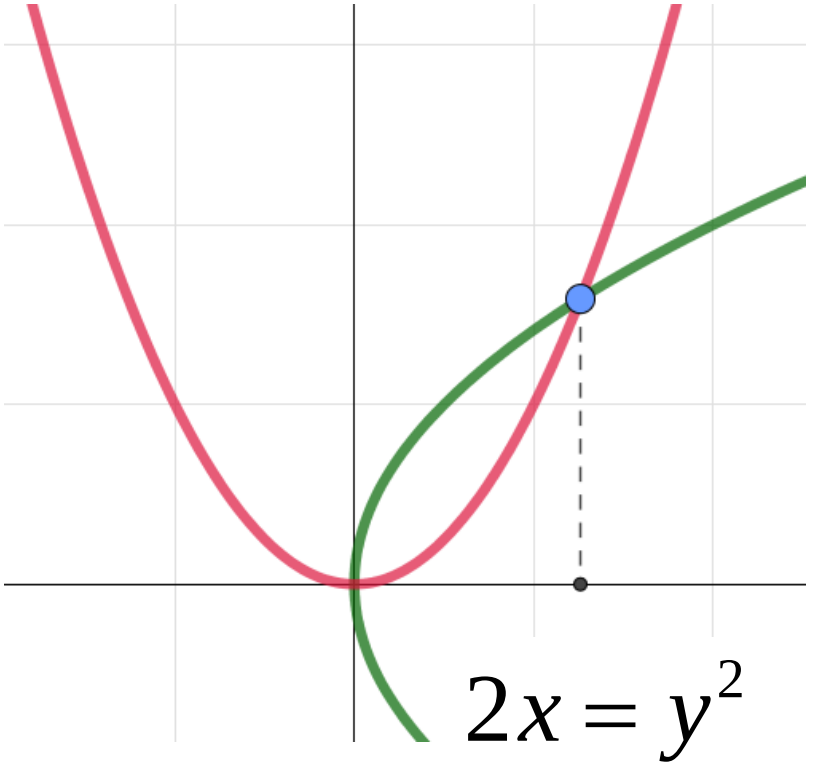
$$\begin{cases} 2x = y^2 \\ y = x^2 \end{cases}$$

Activity

15:30

Which one did I drew this way?

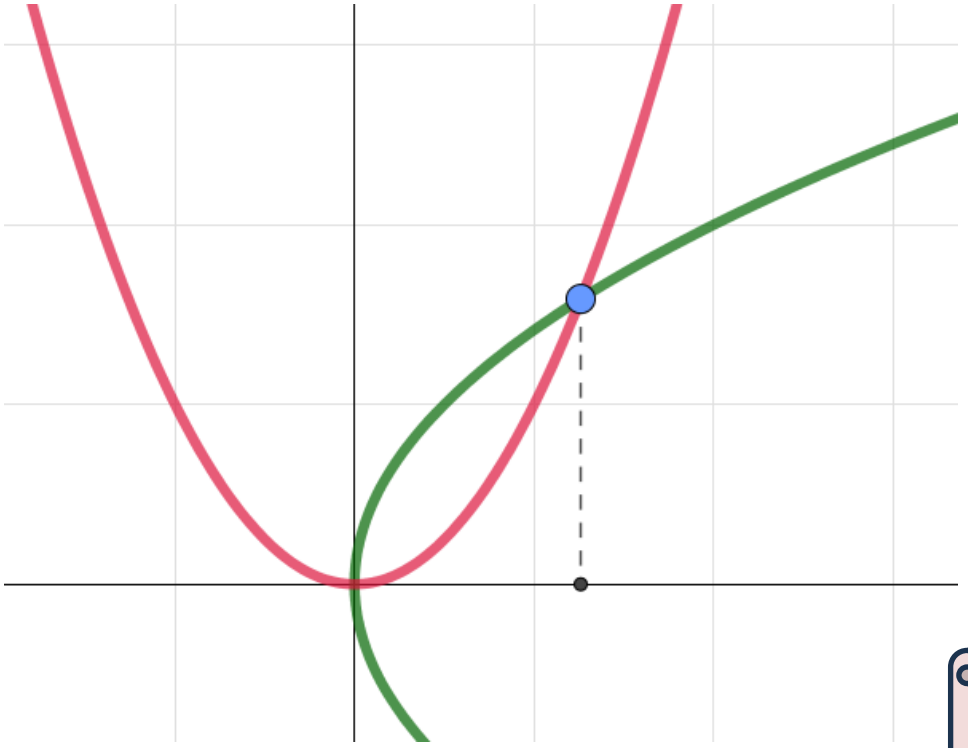


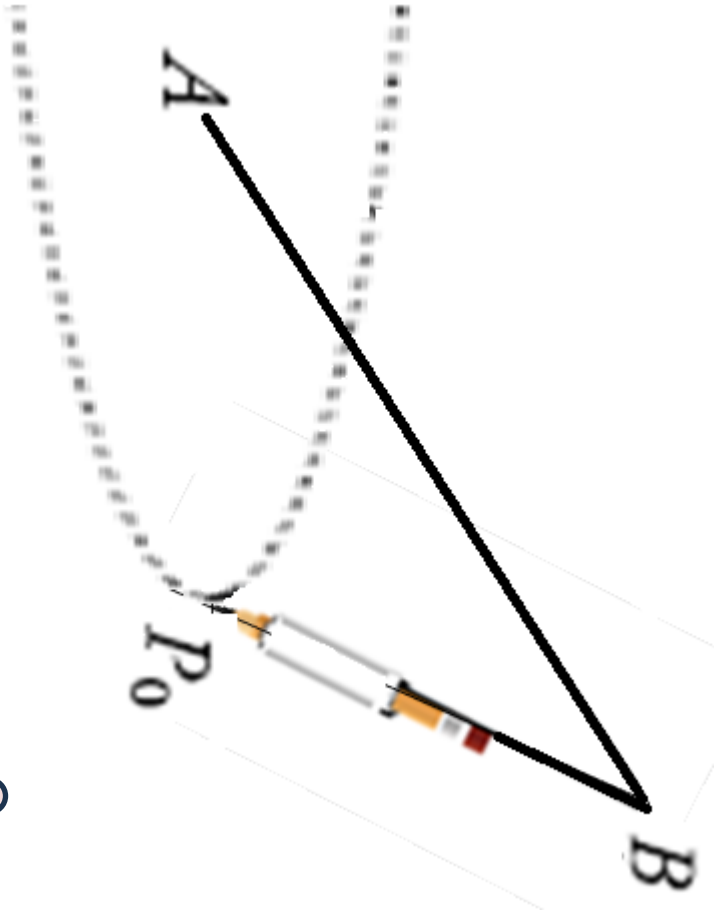


Some help

How to draw the other parabola? What top angle? What cutting angle?

$y = x^2$

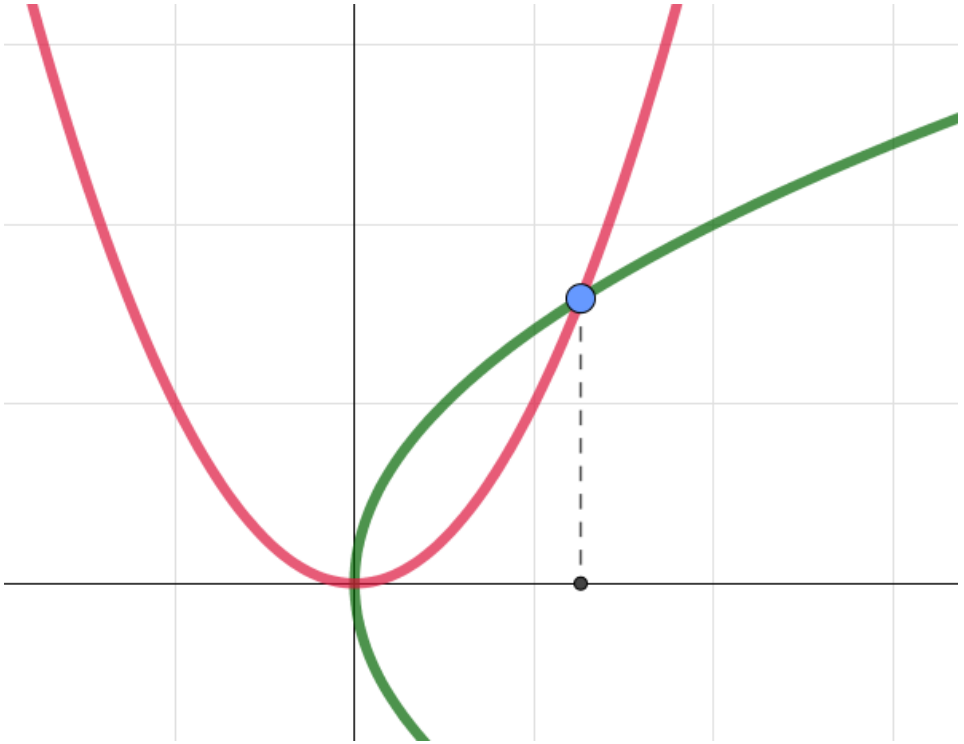


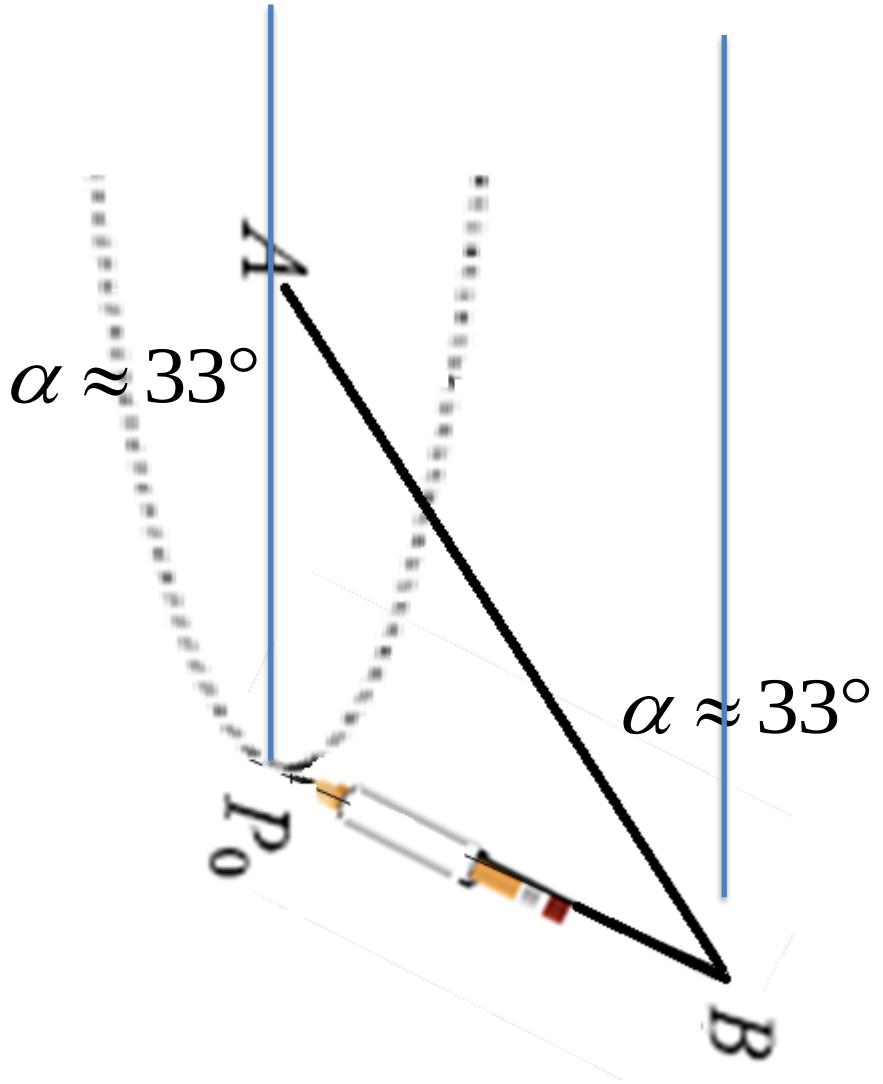


Activity

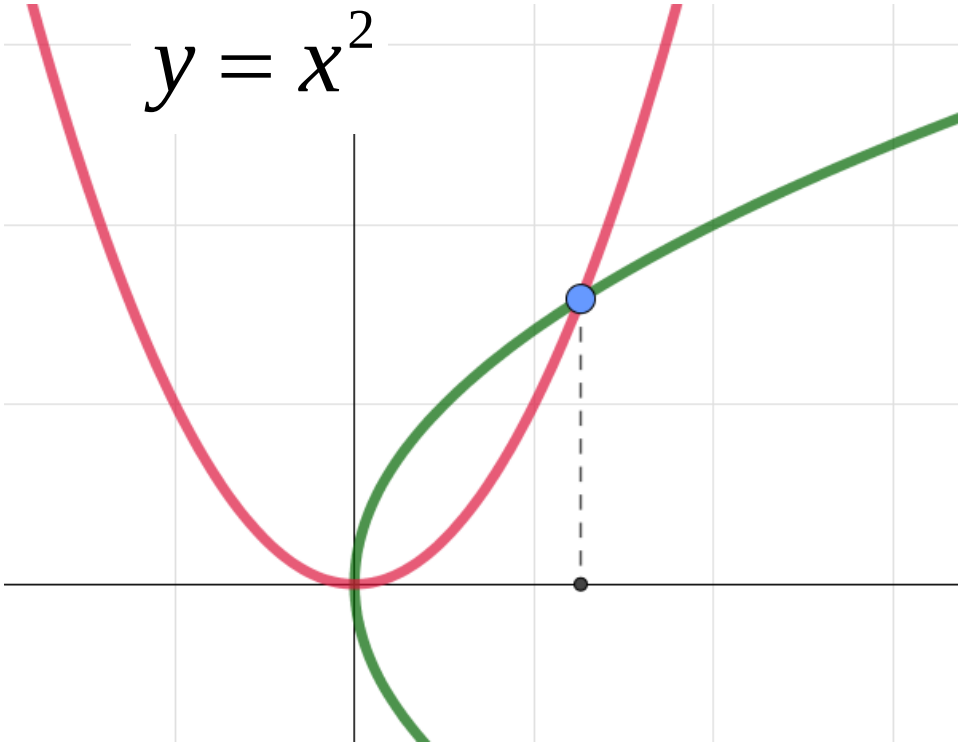
Trace the other parabola

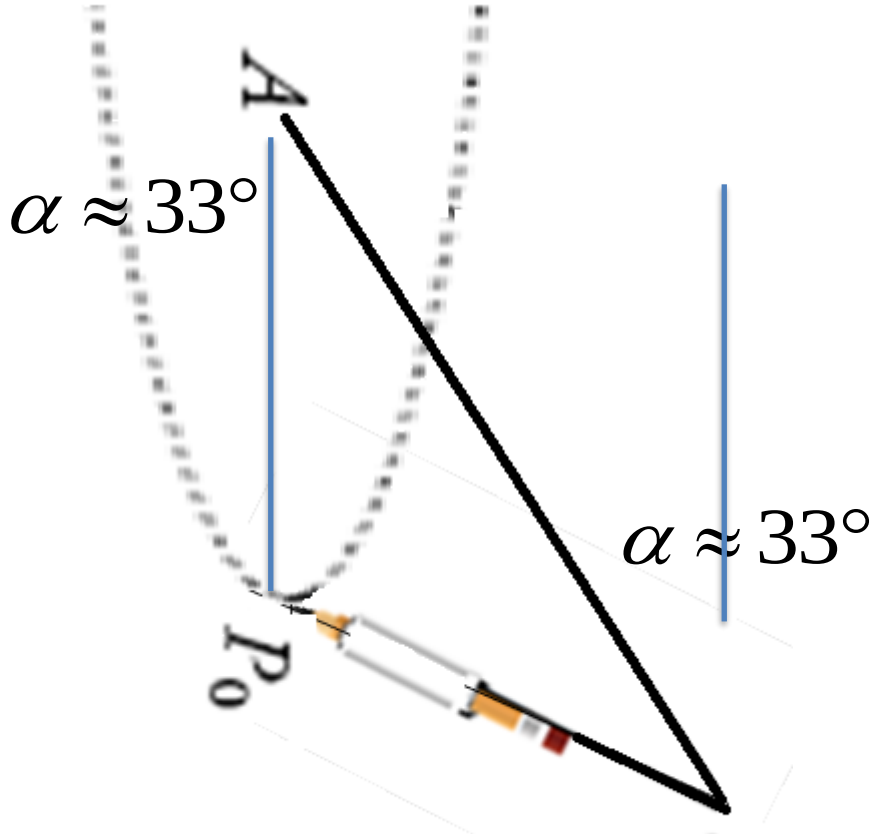
$y = x^2$





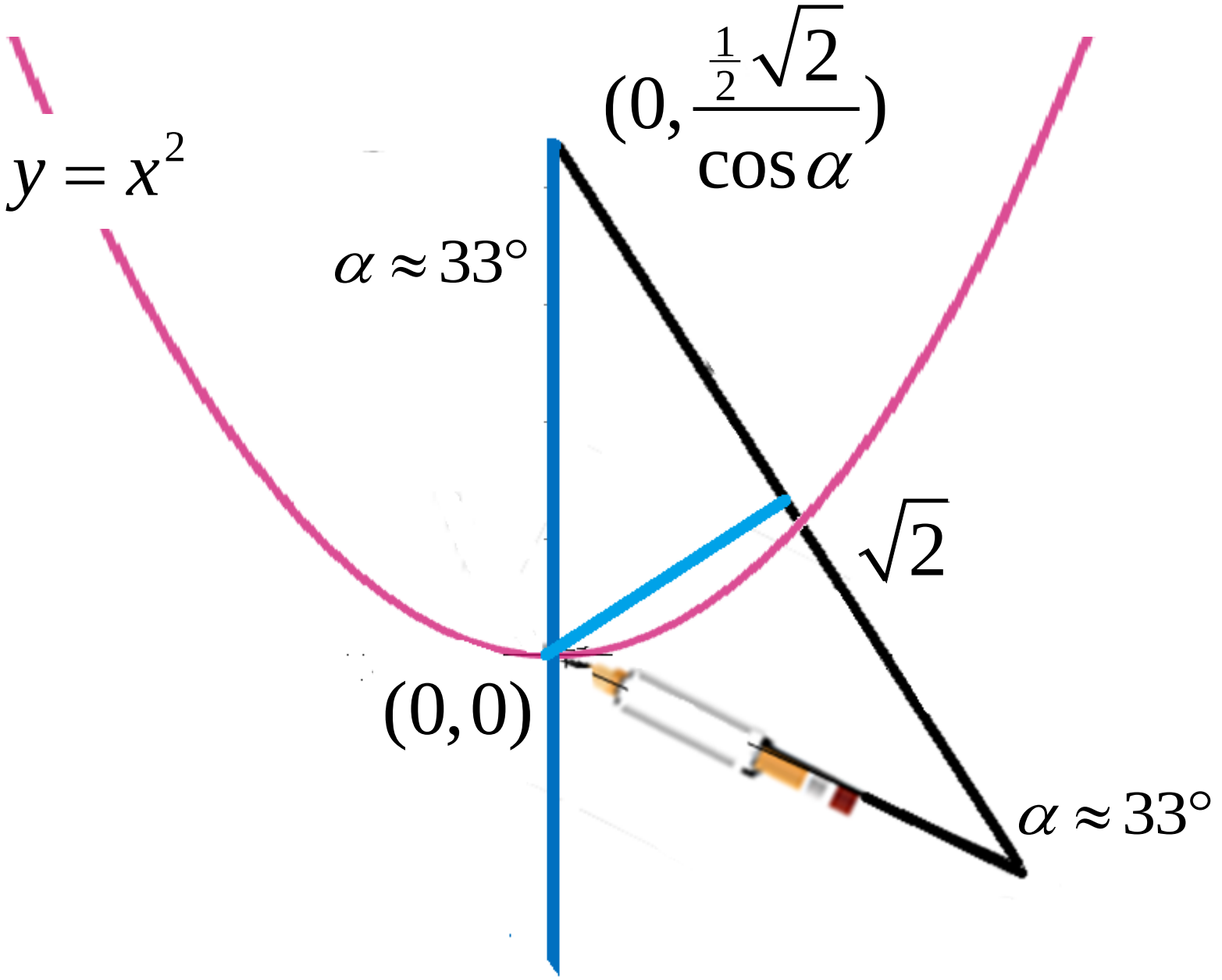
A formula to compute the angle



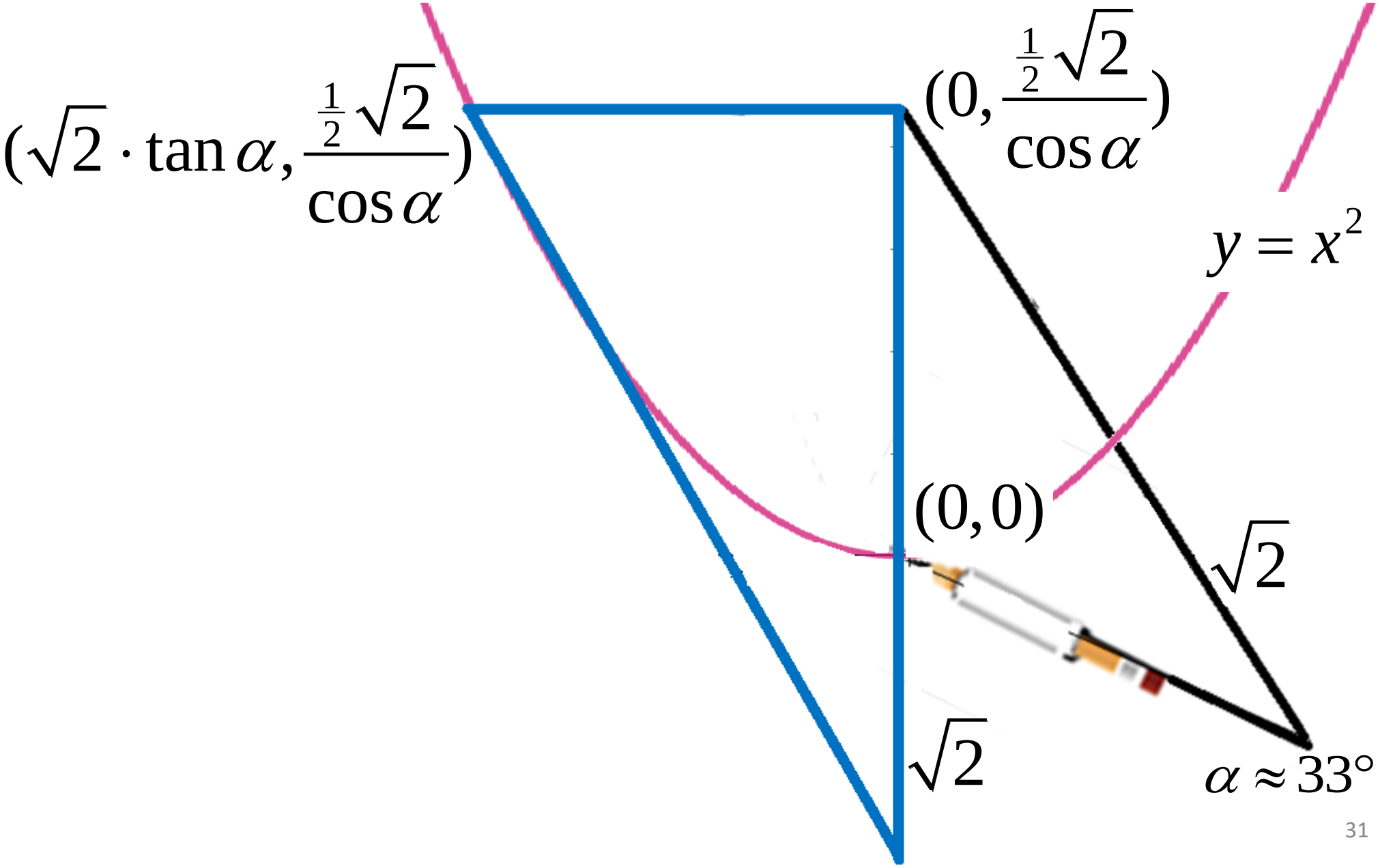


$$\begin{cases} v = AB = \sqrt{2} \\ k \cdot y = x^2 \end{cases} \qquad k = 1 \qquad \cos\alpha = \frac{\sqrt{\frac{1}{16}k^2 + v^2} - \frac{1}{4}k}{v}$$

Acute angle perfect compass
Finding coordinates



Finding coordinates



Solve the equation and compute the angle

$$(\sqrt{2} \cdot \tan\alpha, \frac{\frac{1}{2}\sqrt{2}}{\cos\alpha})$$

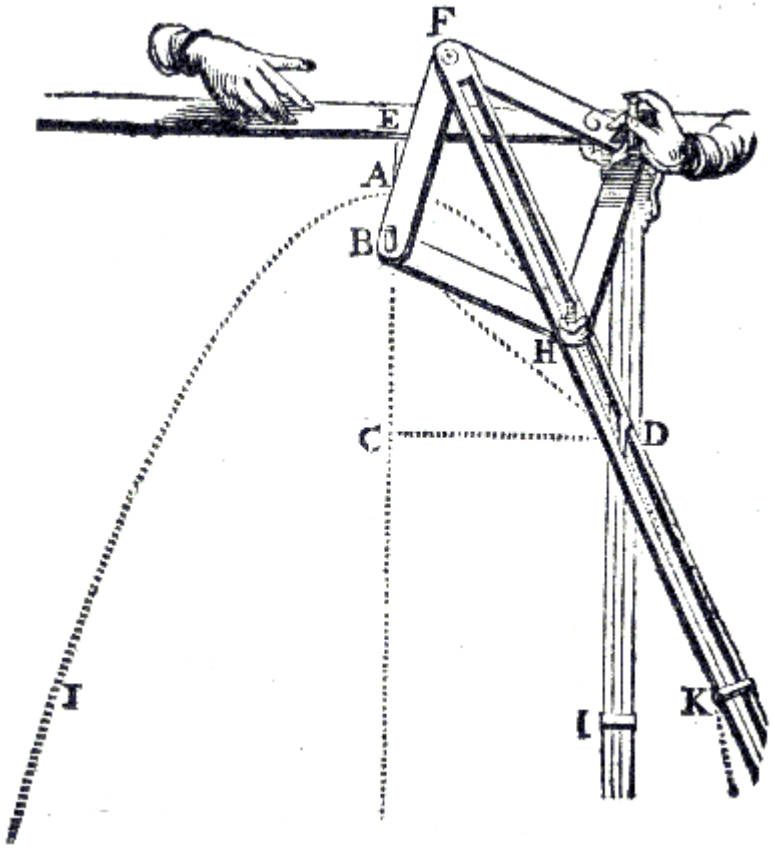
$$y = x^2$$

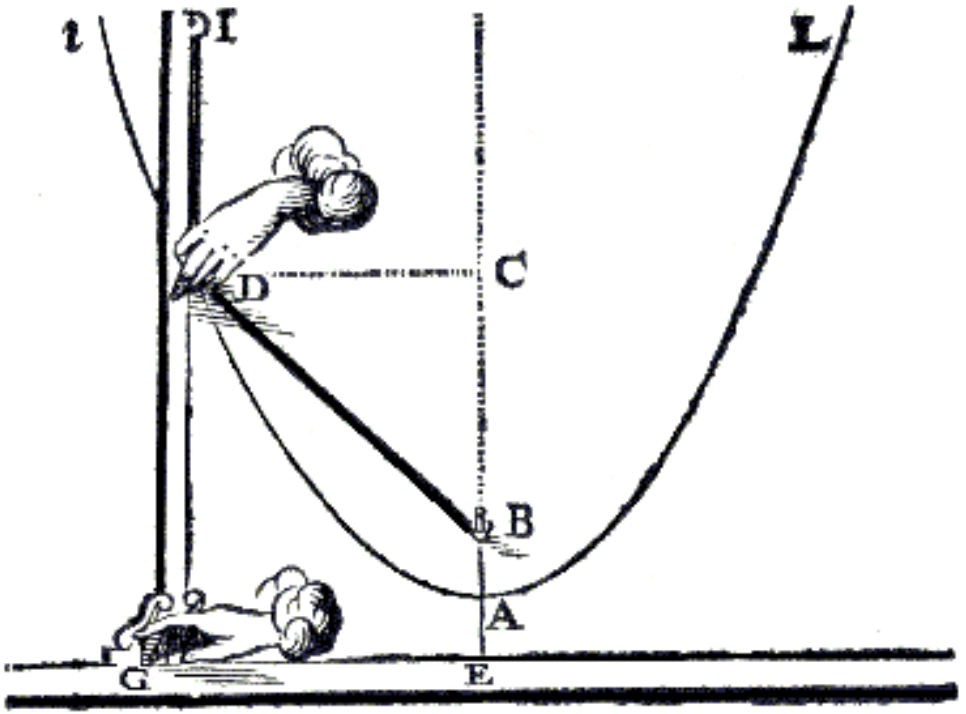
Activity

In general

$$\begin{cases} v = AB \\ k \cdot y = x^2 \end{cases} \qquad \cos\alpha = \frac{\sqrt{\frac{1}{16}k^2 + v^2} - \frac{1}{4}k}{v}$$

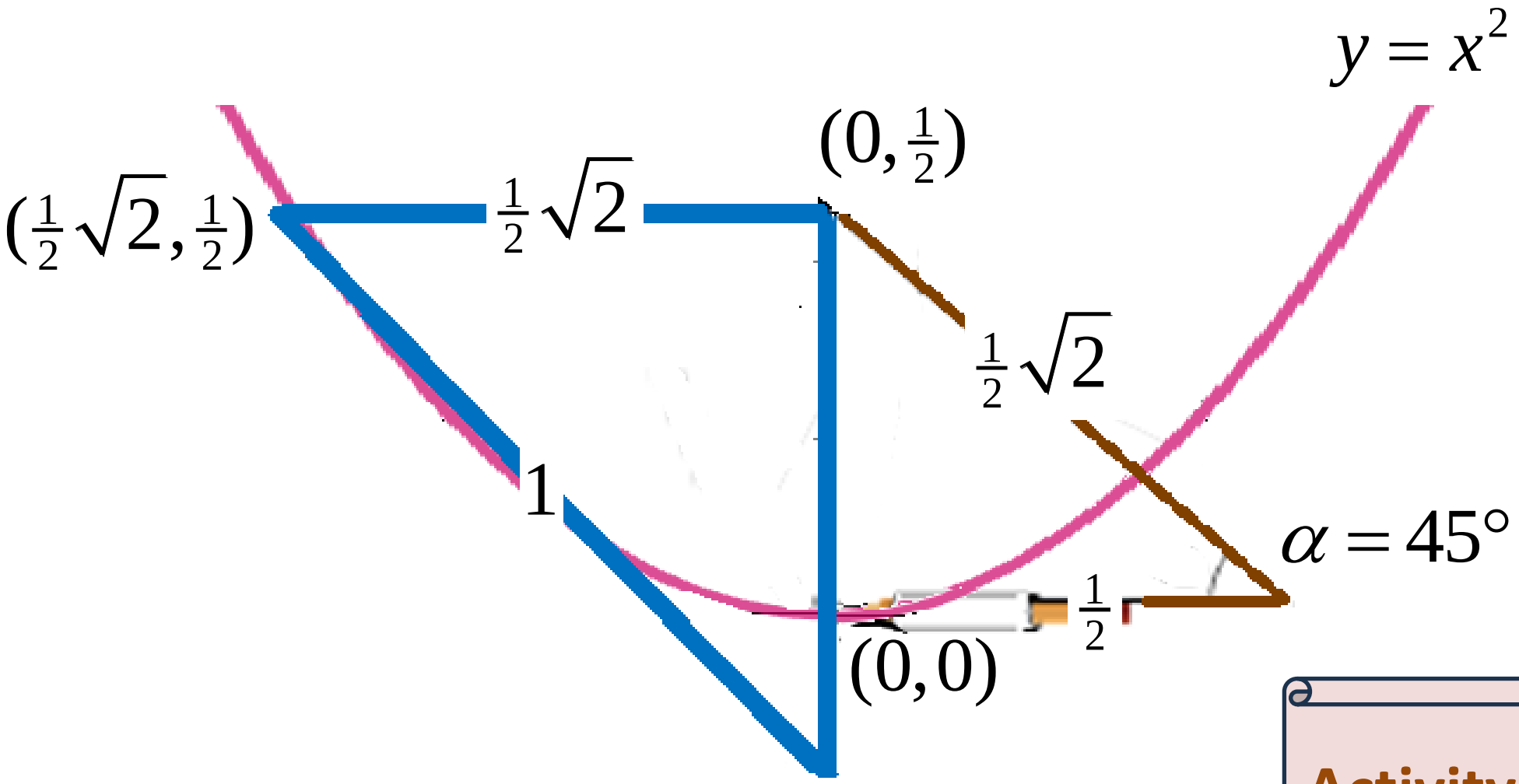
Frans van Schooten
Alternatives for tracing parabolas





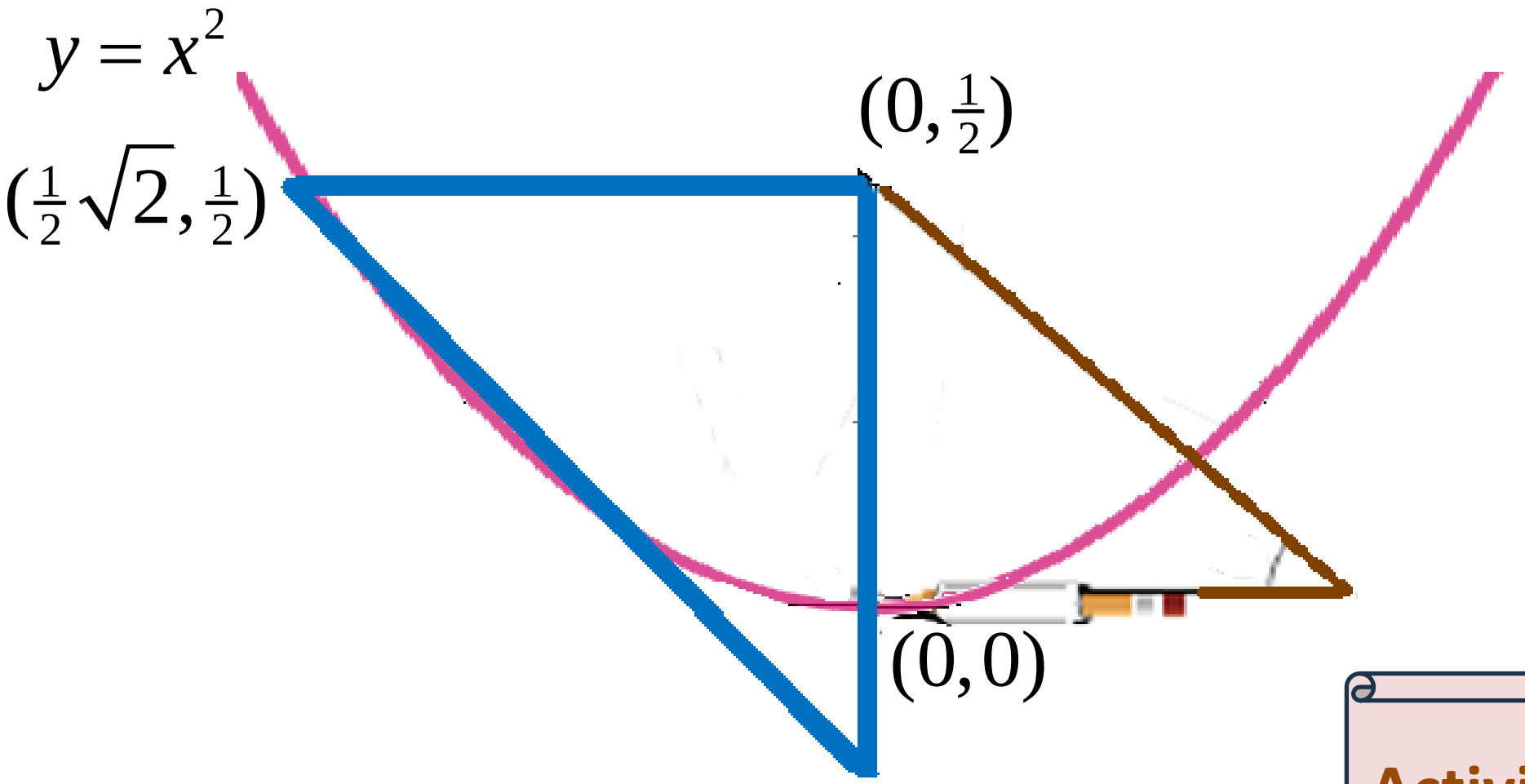
https://fransvanschooten.nl/fvs_tekeningen4.htm

Another right angled perfect compass Check this alternative



Activity

Another right angled perfect compass Check this alternative



Activity

15:40

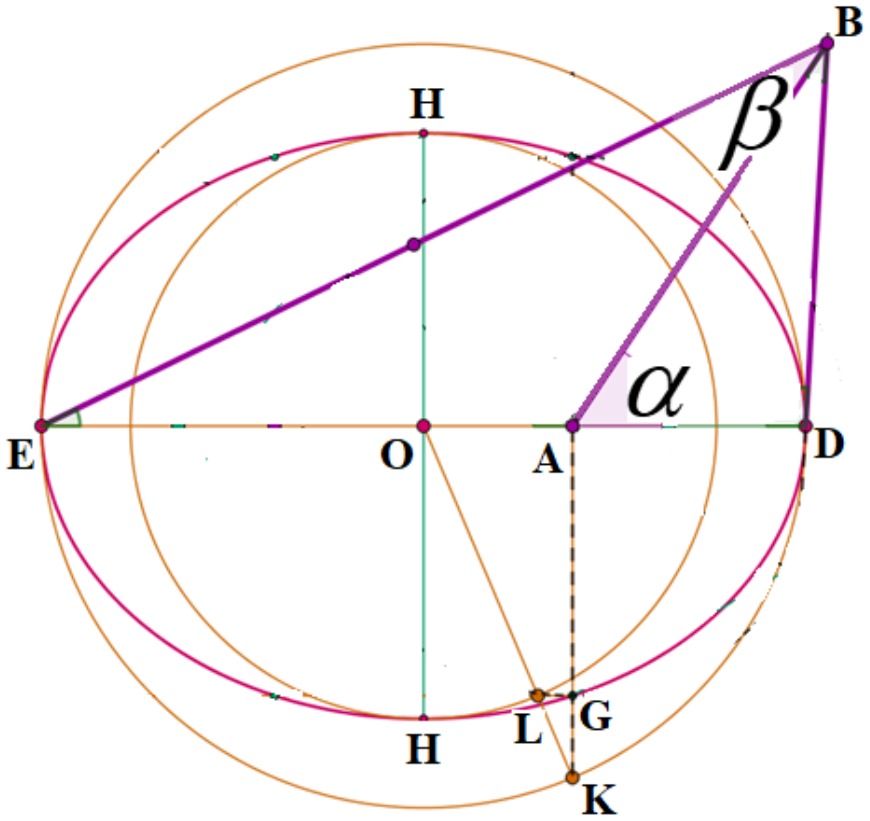
Questions

- **Which instrument would you use to draw parabola?**
  - One universal perfect compass
  - Two dedicated right angled perfect compasses
  - Ruler constructions in the way of Frans van Schooten

Discussion

15:40

Given the angles of the perfect compass, computing the diameters of the ellipse is easy



Apply the sine rule

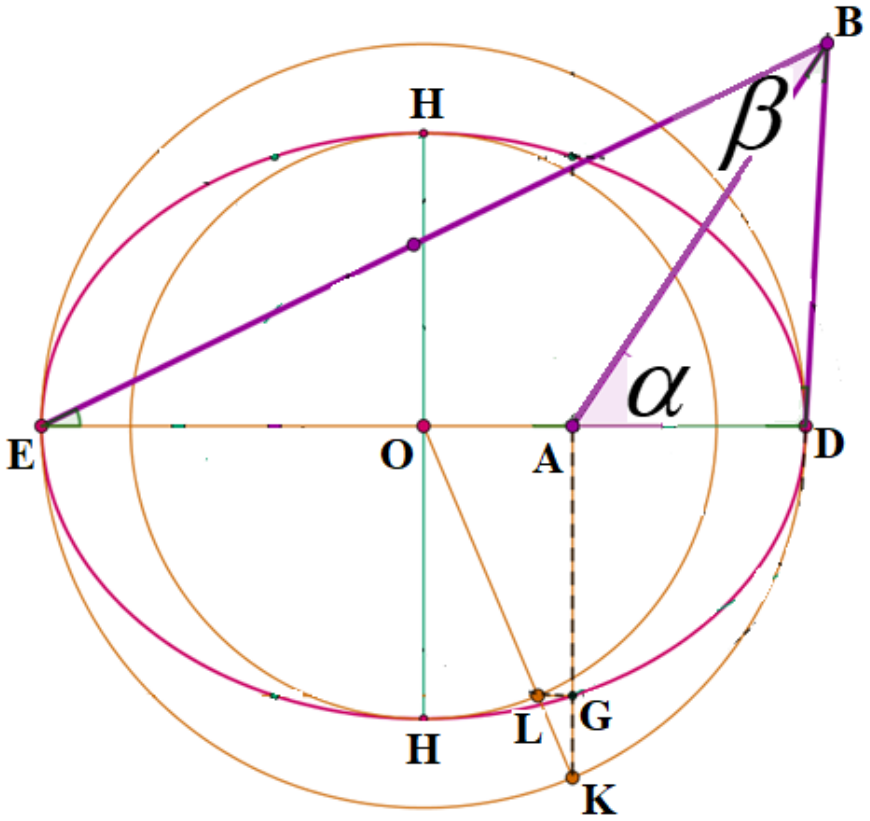
$$\frac{DO}{AB} = \frac{\sin\alpha \cdot \sin\beta \cdot \cos\beta}{\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)}$$

$$\frac{HO}{AB} = \frac{\sin\alpha \cdot \sin\beta}{\sqrt{\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)}}$$

$$\frac{DO}{HO} = \frac{\cos\beta}{\sqrt{\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)}}$$

Activity

Given the angles of the perfect compass, computing the diameters of the ellipse is easy



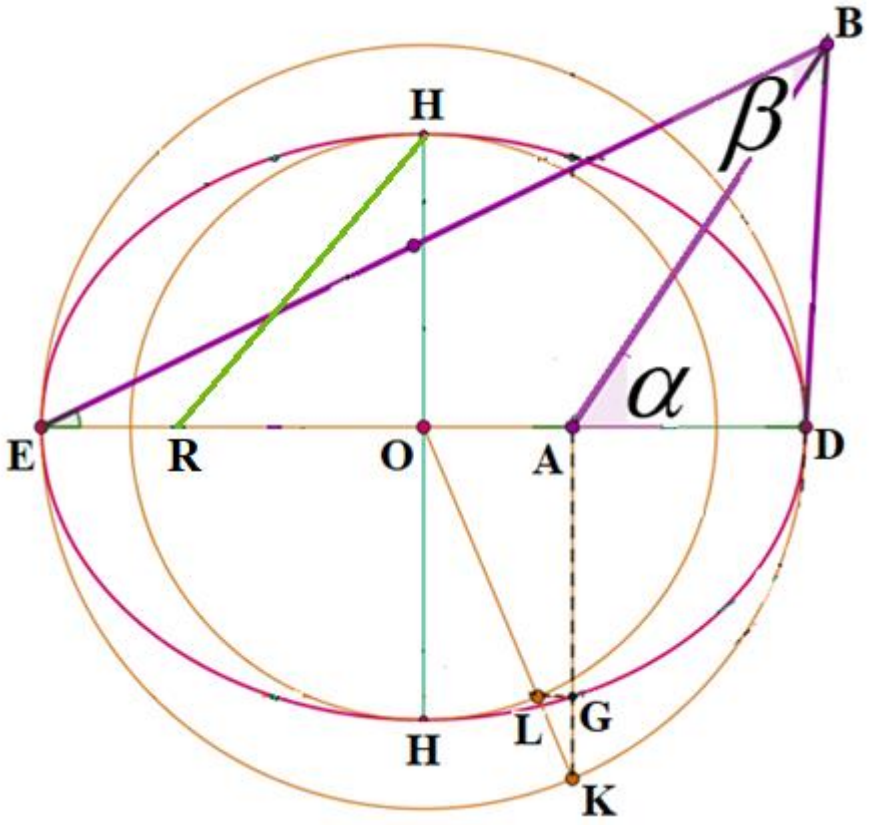
Eulid 4.13:

Square root in a circle

$$k = \frac{HO^2}{AB \cdot DO} = \sin\alpha \cdot \tan\beta$$

Activity

Given the angles of the perfect compass, computing the diameters of the ellipse is easy

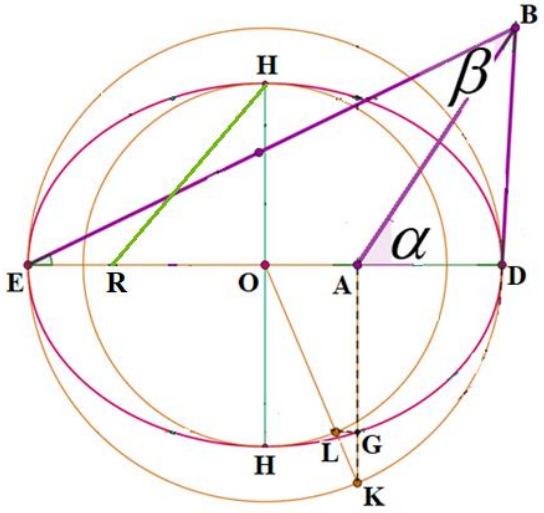


Just algebra

$$\varepsilon = \frac{OR}{DO} = \sqrt{1-\left(\frac{HO}{DO}\right)^2} = \frac{\cos\alpha}{\cos\beta}$$

Activity

Given the size of the ellipse, computing the angles of the perfect compass is hard work



$$\frac{DO}{AB} = \frac{\sin\alpha \cdot \sin\beta \cdot \cos\beta}{\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)}$$

$$\varepsilon = \frac{OR}{DO} = \sqrt{1-\left(\frac{HO}{DO}\right)^2} = \frac{\cos\alpha}{\cos\beta}$$

$$\frac{HO}{AB} = \frac{\sin\alpha \cdot \sin\beta}{\sqrt{\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)}}$$

$$k = \frac{HO^2}{AB \cdot DO} = \sin\alpha \cdot \tan\beta$$

$$\frac{DO}{HO} = \frac{\cos\beta}{\sqrt{\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)}}$$

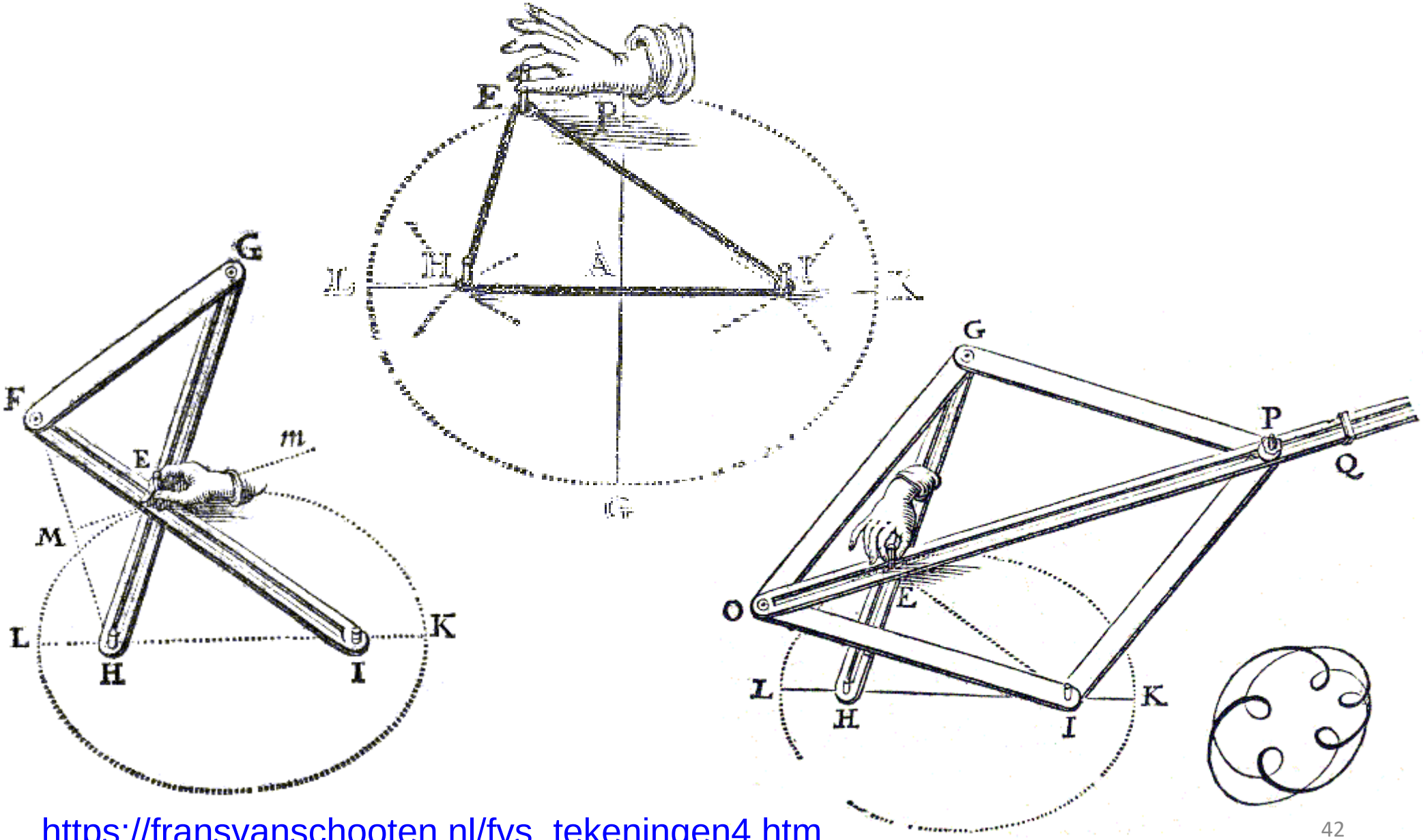
$$\cos^2\beta = \frac{(1+\varepsilon^2+k^2) - \sqrt{(1+\varepsilon^2+k^2)^2 - 4\varepsilon^2}}{2\varepsilon^2}$$

$$\cos^2\alpha = \frac{(1+\varepsilon^2+k^2) - \sqrt{(1+\varepsilon^2+k^2)^2 - 4\varepsilon^2}}{2}$$

Activity

15:50

Frans van Schooten Jr (1615 - 1660)
Drawing Ellipses



https://fransvanschooten.nl/fvs_tekeningen4.htm

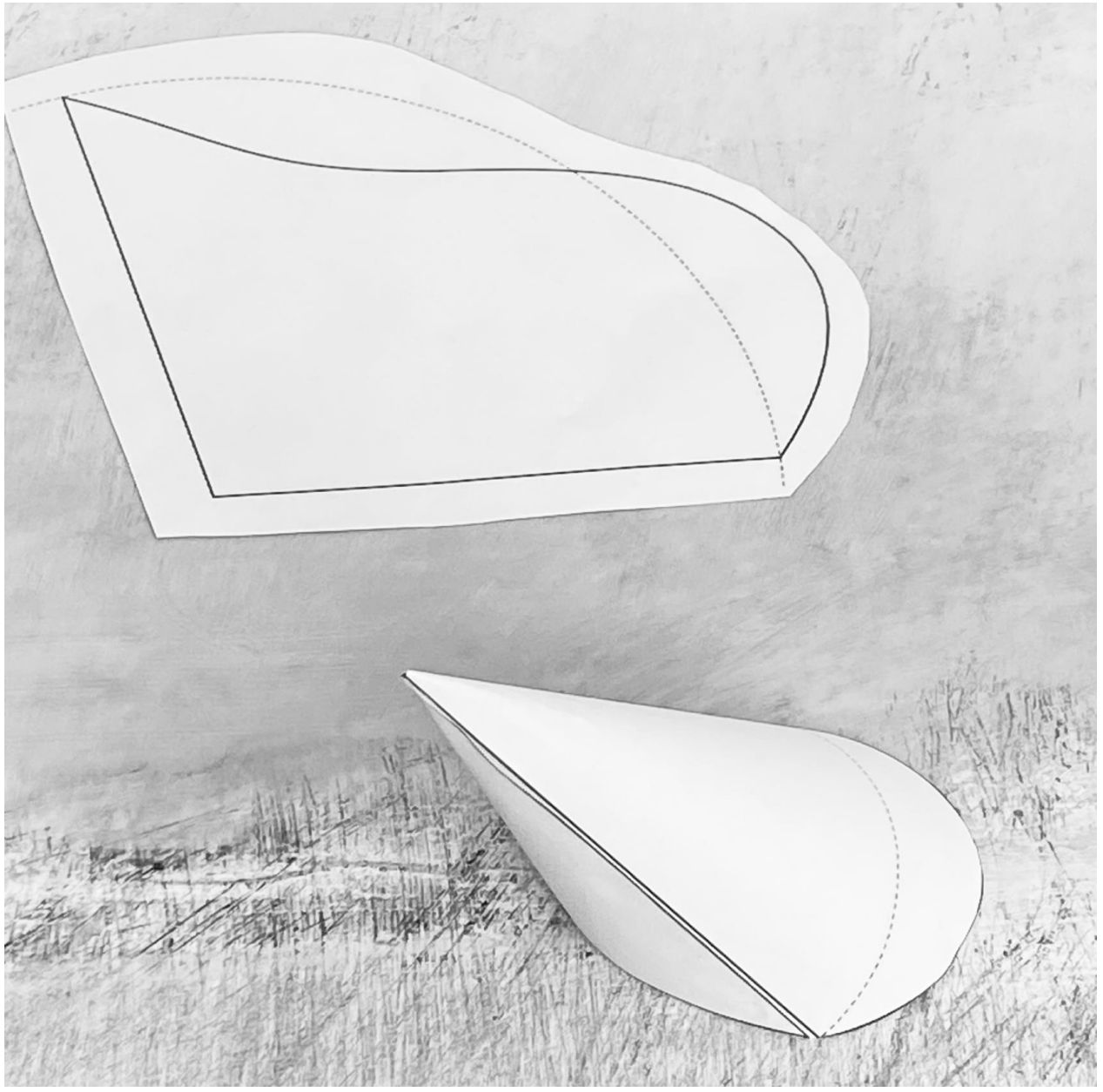
Questions for drawing ellipses

- **Which instrument would you use to draw ellipses?**
  - Universal perfect compass
  - Dedicated 45° perfect compasses
  - Gardeners ellips construction
  - Ruler constructions in the way of Frans van Schooten

Questions for mathematicians

- **Which instrument would you like to teach when dealing with parabola and ellipses?**
  - Universal perfect compass
  - Dedicated 45° perfect compasses
  - Ruler constructions in the way of Frans van Schooten

Henk Hietbrink
Folding Ellipses



Thank you for your attention