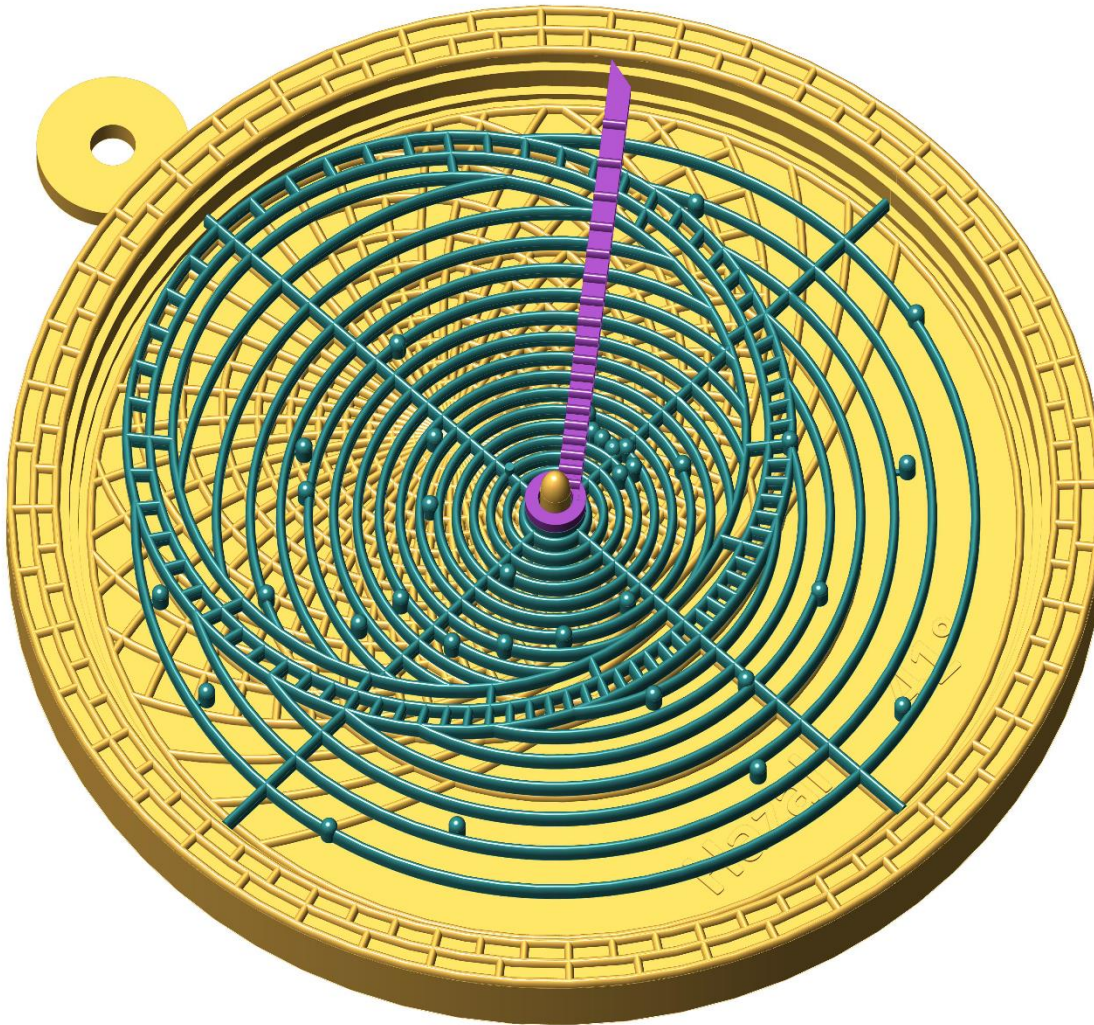


# What do you see on your 3D-printed Astrolabe?



This image shows the main parts of the front of an astrolabe. Usually, astrolabes are completed with multiple plates for different latitudes. Yours has only one for your latitude.

The limb is then the protruding edge within which a collection of plates is stored. The bottom of the round box is called the mater. Specific mathematical diagrams may be drawn on the mater. In yours, the plate for your latitude has been printed. It doesn't have removable discs. The front of your astrolabe has three parts:

- The main body comprising of limb and mater and plate (this is the yellow object with a long pin)
- Rete (blue set of circles with tiny spots for the stars)
- Ruler (the rotatable purple bar for precise measurement of directions and declinations)

The back has only two items

- Conversion diagram between position in the zodiac and date in the year.
- A ruler (and sometimes a complete alidade with sights)

All details will be described separately.

## About the astrolabe

The astrolabe is a mechanical instrument to solve problems related to the position of sun and stars, in the past, at the moment and in the future. For example, you can determine when the sun rises on your birthday, or at what time the sun goes down on your mothers wedding day. The astrolabe tells you how high the sun is in the sky at 11:00 on December 30. You can also determine what time it is when the star Betelgeuse in the constellation Orion is just  $20^\circ$  above the horizon. With Islamic astrolabes you can determine the times of prayer.

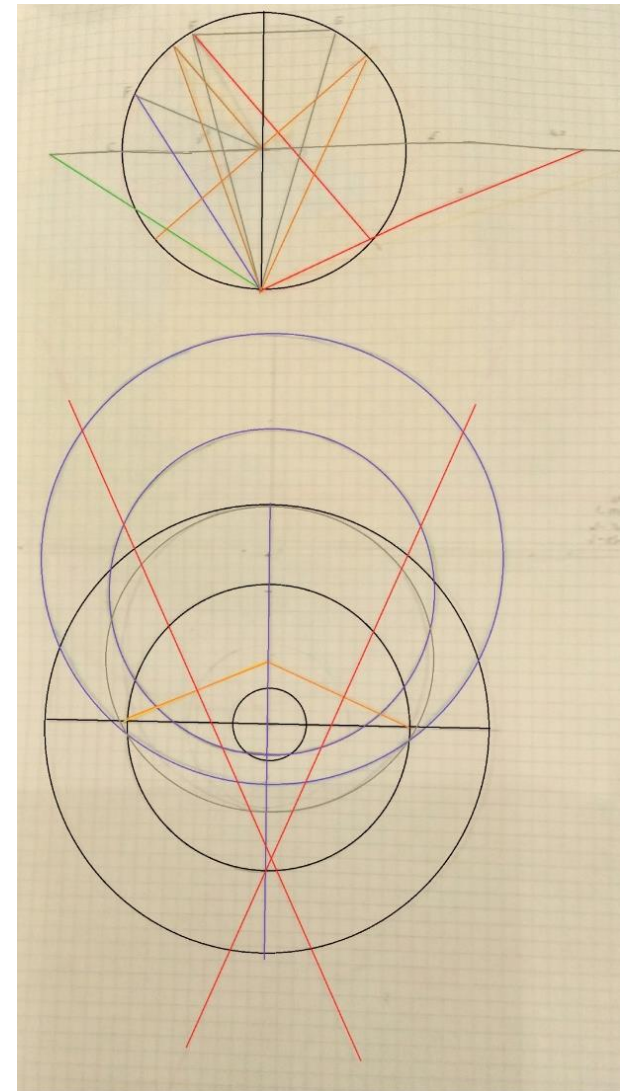
Nowadays you can find the answers to these kinds of questions on the internet or with a handy app on your phone, but when there were no computers, the astrolabe gave a definite answer, at least if you had a good one that matched your location and if you were an expert in manipulating and reading off.

The astrolabe is an old instrument. In Greek times, the necessary mathematical knowledge was available before the beginning of the era. The first descriptions date from the sixth century after the era. In the Arabic language area it is written about from the tenth century. There are copies from the same time in museums. Four centuries later, the first astrolabes appear in Western Europe.

Today, the astrolabe is a historical instrument and a great tool for understanding the mathematics behind stereographic projection and spherical trigonometry.

An astrolabe does not show the orbit of the planets. An astrolabe shows the orbit and position of the sun and of the fixed stars. Therefore, the instrument is of little value to an astrologer who wants to draw a horoscope related to the conjunction of Mars and Venus. The astrolabe is not intended for navigation and certainly not for navigation on a ship at sea. The mariner's astrolabe is an incomparably different instrument that has only its name in common. An astrolabe is not intended for calculating latitude. On the contrary, each astrolabe is designed for a specific latitude. Some astrolabes have lines indicating prayer times. However, it is more easy to use a sundial in order to determine prayer times related to the length of the shadow.

Although the back of an astrolabe may contain more diagrams, this document focuses on the front of the astrolabe.



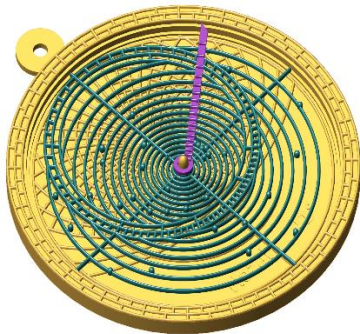


## Introduction

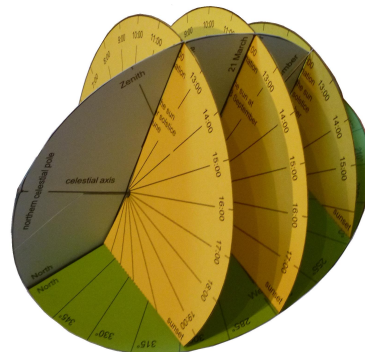
This document describes most of the lines you see on the front of the astrolabe. It is intended for memorizing all the lines and circles as a first step in understanding the design of an astrolabe. A second document describes the geometry and trigonometry of all the lines and circles. That document also contains exercises.

This document mentions a 3D printed astrolabe; the other document mentions a laminated paper astrolabe of the same design. The illustrations of the 3D printed astrolabe design are pretty good for lectures, but the 3D printed astrolabe itself is less accurate than the paper model because an ink printer can draw much finer details than a 3D printer ever can.

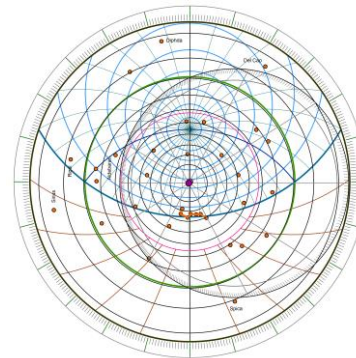
This document also mentions a paper model of the most important planes for studying sundials and astrolabes.



3D printed astrolabe

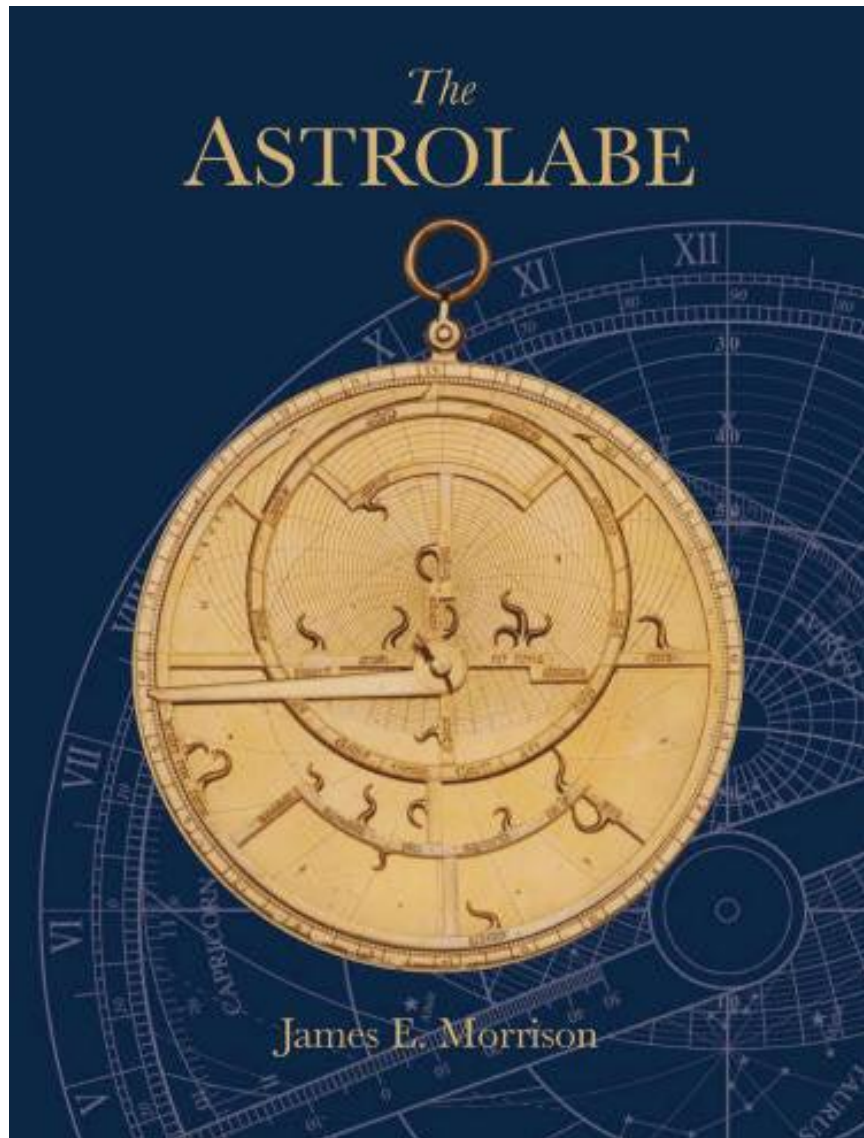


Paper model of the planes



Laminated paper astrolabe

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## Morrison: the Astrolabe

For the past ten years, this comprehensive book by Morrison has been my main reference work. Meticulously, every part of the astrolabe is explained with clear illustrations, necessary theory and with many examples. Morrison passed away. His website is archived. Second hand copies are collectioners items. The reprint had a too high price. Illegal copies can be downloaded from the internet.

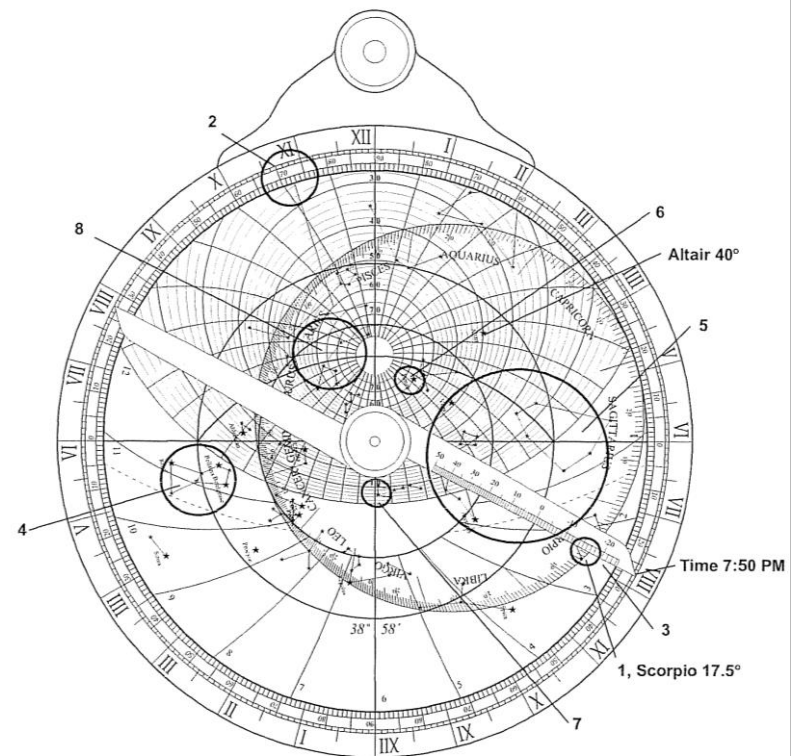
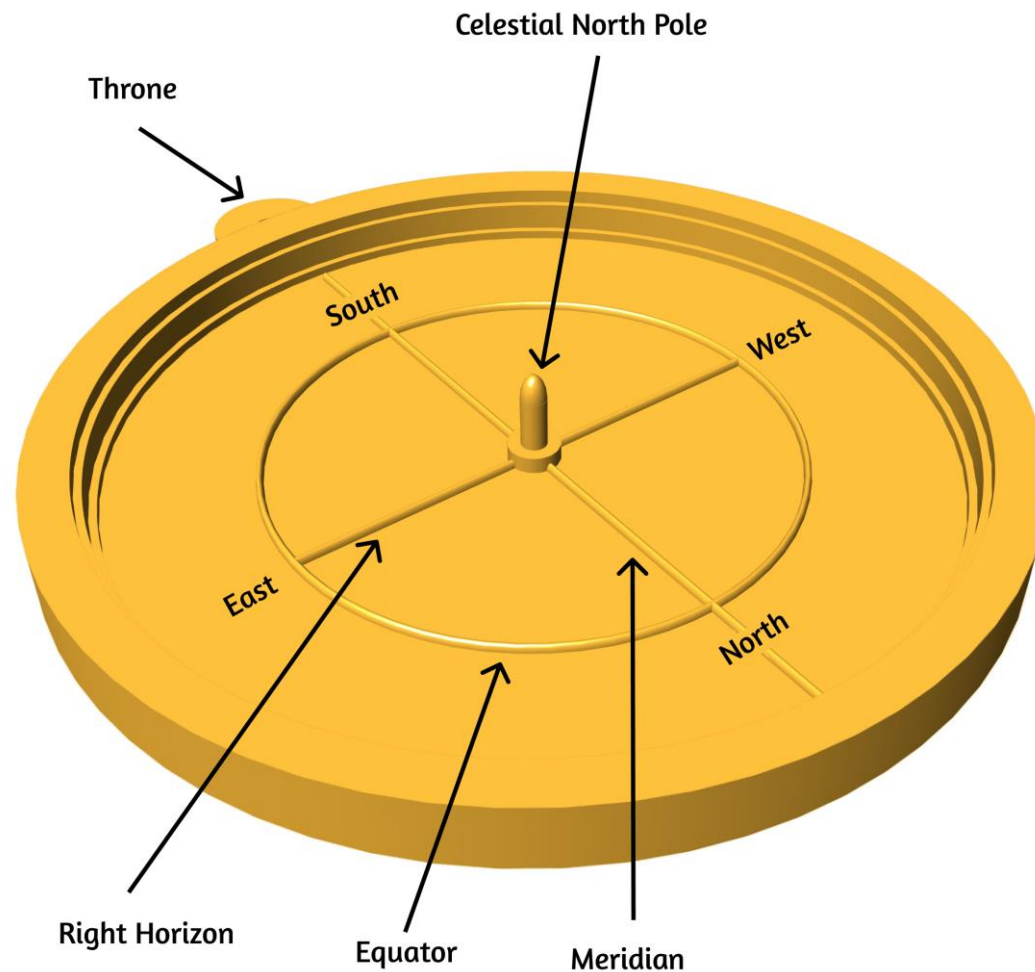


Figure 1-16. Astrolabe set for November 9, Altair at 40°



## Equatorial plane

The limb and plate deal with the celestial equator.

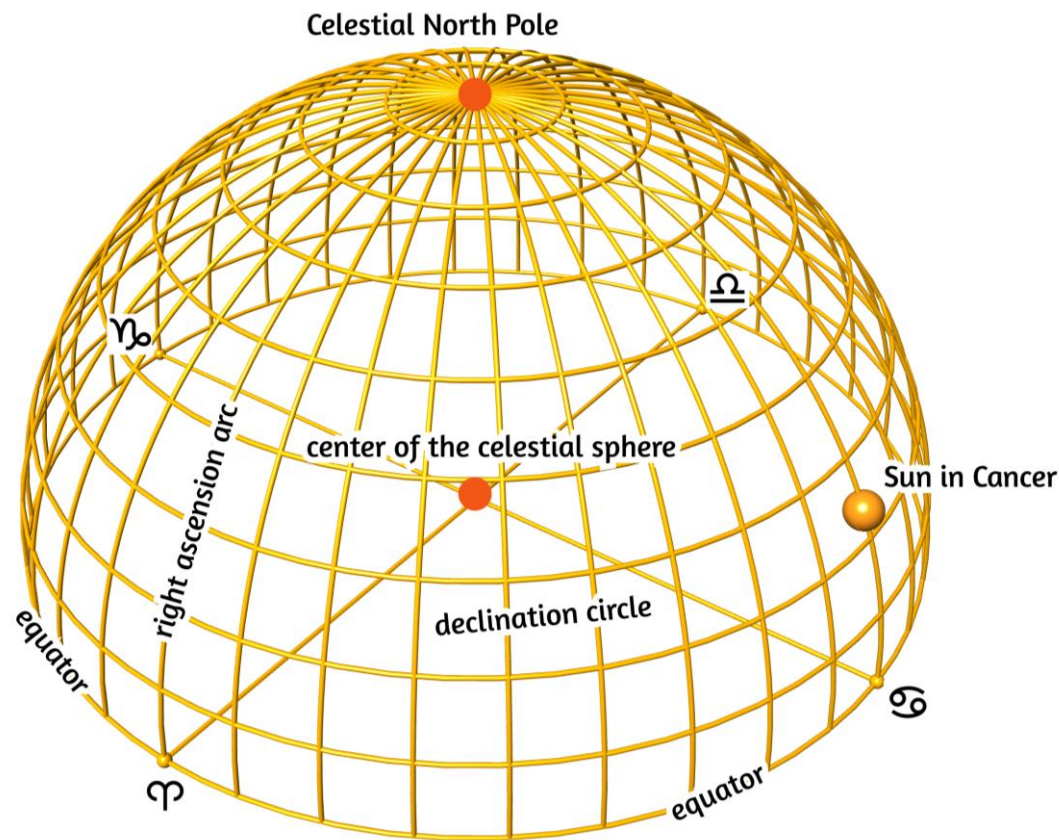
Holding your astrolabe at an angle upward, point the throne to the south and direct the pin toward the Pole Star for a natural orientation of the sun's movement along the sky.

In this way you keep the astrolabe parallel to the equatorial plane. The equatorial plane is perpendicular to the earth's axis of rotation (from the celestial north pole to the celestial south pole). There is no celestial east or west. For now, it is important to underline the distinction between celestial north and north at the horizon.

The front plate is the projection of a sphere. The center point is the projection of the celestial north pole.

- Major circle:
  - Equator
- Special point:
  - Celestial North Pole
- Compass directions:
  - North, East, South, West
- Special lines
  - Meridian (North to South)
  - Right Horizon(East to West)





## Northern Hemisphere

The limb and the rete deal with the celestial equator.

Suppose all the stars are fixed on a sphere with the celestial north pole on top. This sphere is like a map. The horizontal grid lines are the circles of declination and the vertical lines are arcs of right ascension.

This hemisphere turns round in 24 hours. So, during a day, an object's declination and right ascension do not change. It is altitude and azimuth that do change, a different coordinate system.

Each celestial body has a position on this sphere. Stars have a fixed position, but the sun moves across the sphere in a special annual orbit, and the planets travel their own course.

For instance, around 20 or 21 March, the sun crosses the equator in the sign Aries ♈, the sun reaches its highest position, 23,4° declination, around 21 June at the summer solstice in the sign Cancer ♋ and its lowest position in December in the sign of Capricorn ♐.

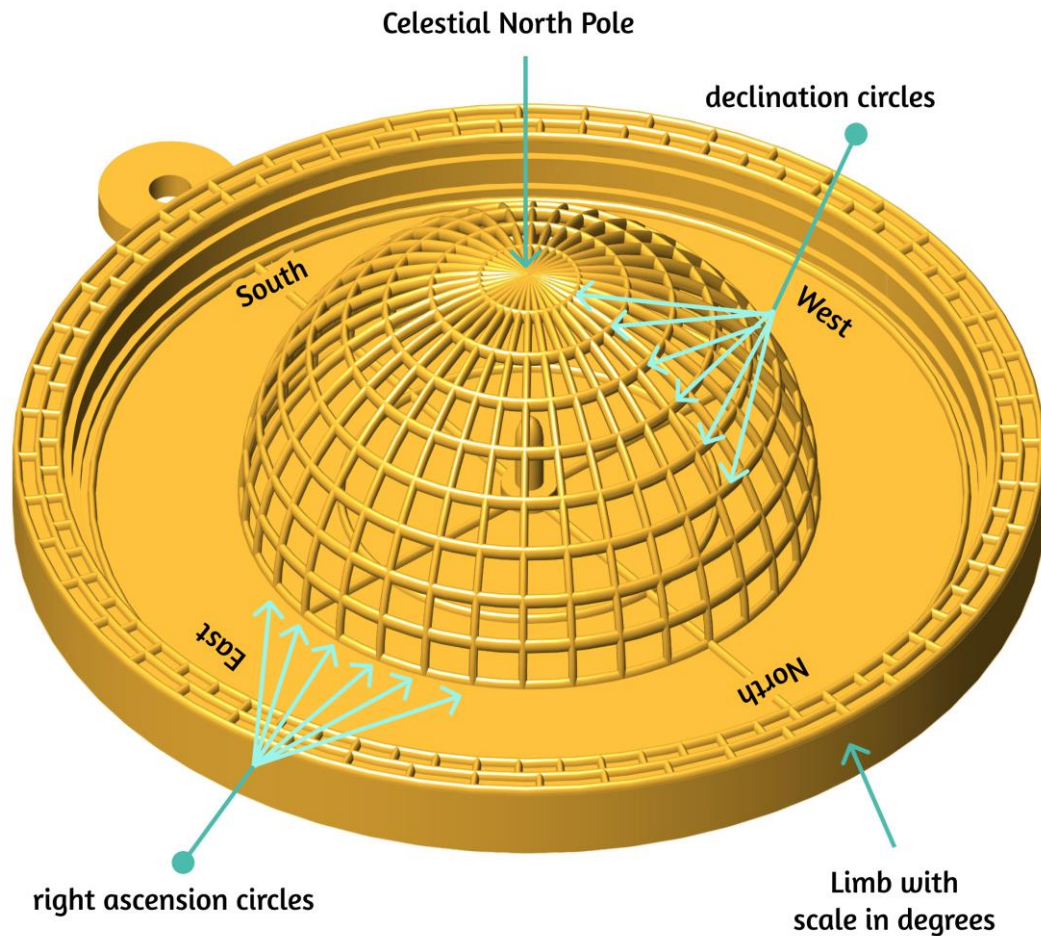
## Northern Hemisphere Projection

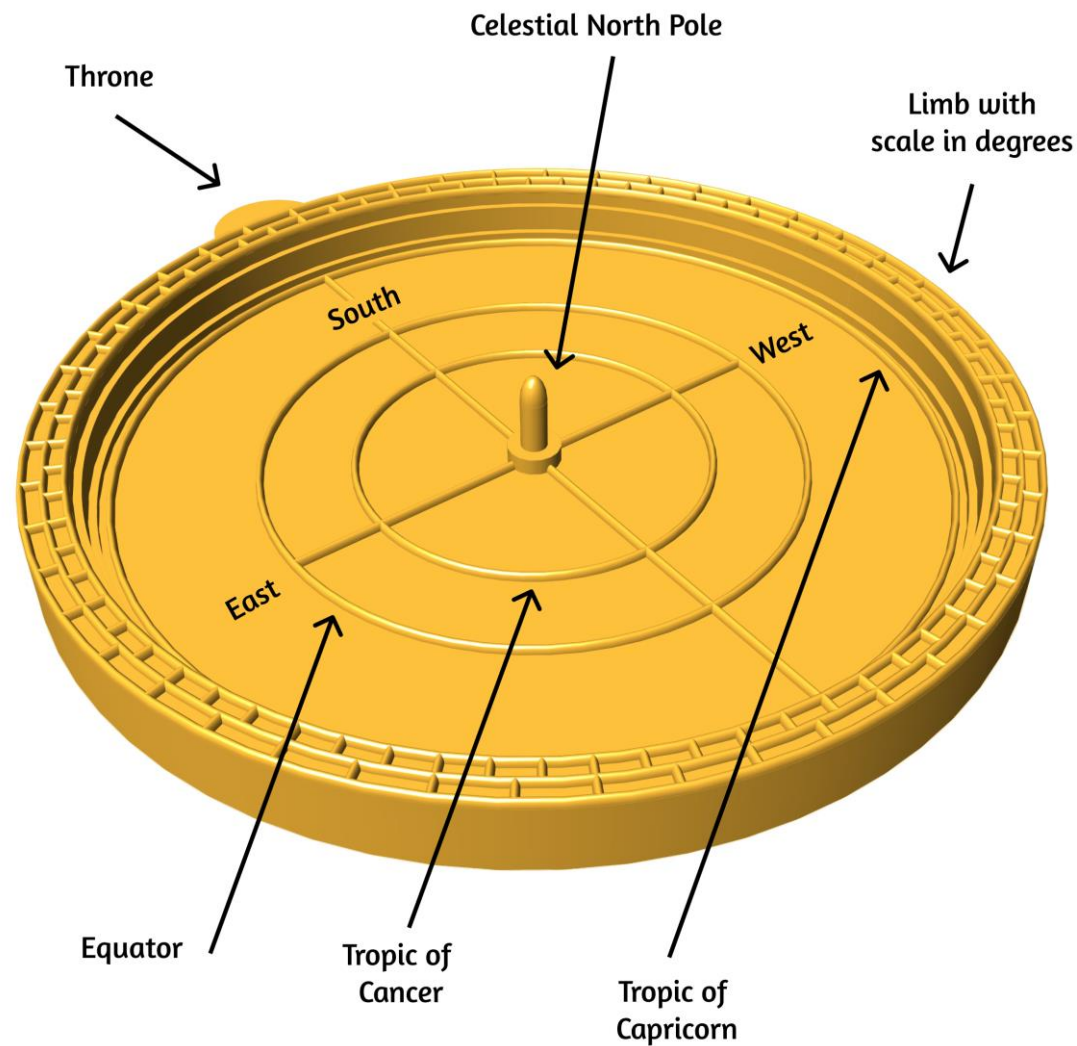
The northern hemisphere is projected onto the rete. The pin is the centerpoint; it is the projection of the celestial north pole.

The circles of right ascension on the hemisphere are large circles on the sphere. Right ascension is measured in degrees along the equator from the sun's position at the March equinox, the so-called first point of Aries or vernal equinox. The division of the limb corresponds to right ascension. The limb is usually divided into (multiples of) degrees.

Declination circles are small circles in the hemisphere, except for the equator which is a large circle. The celestial north pole is a point with  $90^\circ$  declination.

- Compass directions:
  - North, East, South, West
- Special point:
  - Celestial North Pole
- Major circles:
  - Declination circles  
each circle is the set of points at equal declination above the equator
  - Right ascension circles  
each circle is the set of points having the same angle as the meridian circle





## Tropics

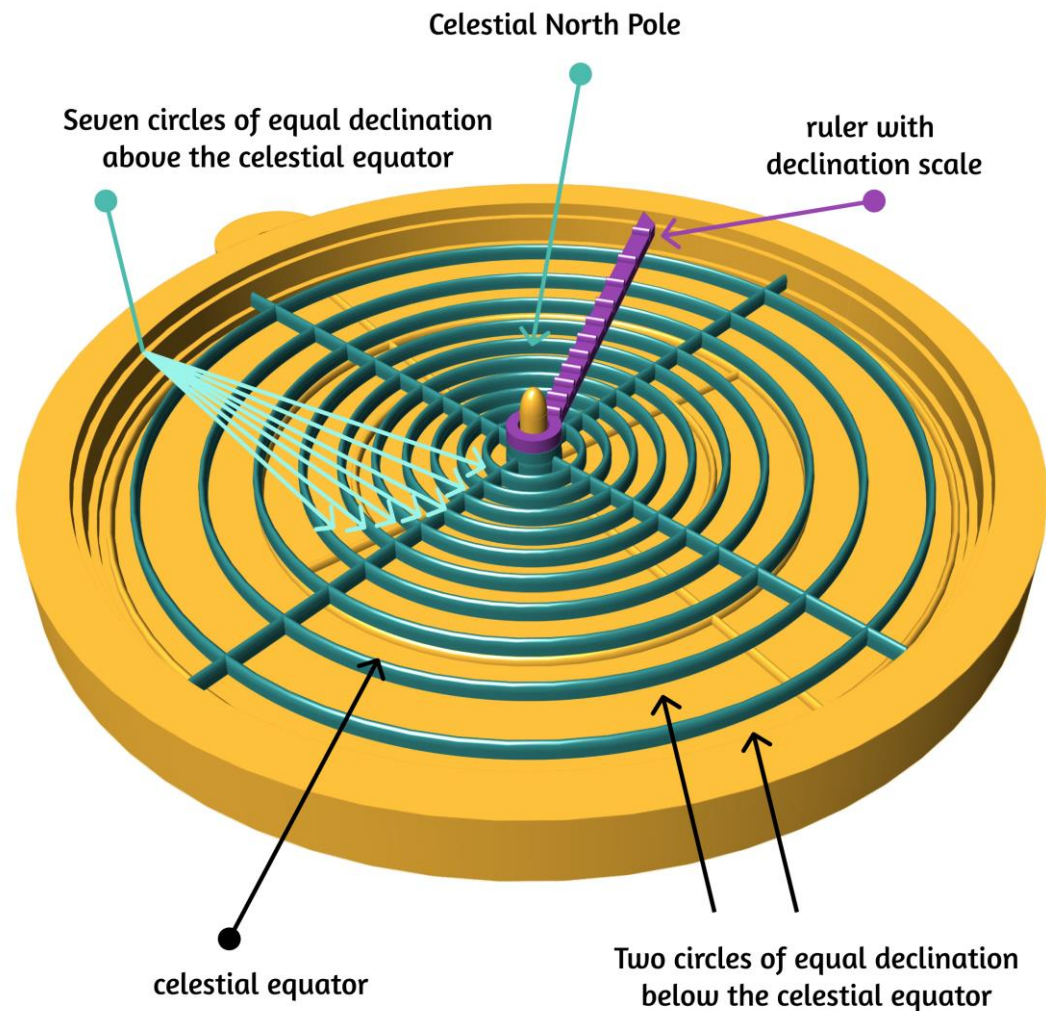
The Tropic of Cancer and the Tropic of Capricorn are declination circles related to the solstices. During the summer solstice, the sun is at the height of the Tropic of Cancer. During the winter solstice, the sun is at the height of the Tropic of Capricorn.

The Tropic of Cancer is north of the equator, the Tropic of Capricorn is south of the equator.

The projection of the Tropic of Cancer has a smaller radius than the equator. The projection of the Tropic of Capricorn is at inner side of the boundary of the astrolabe plate.

- Special point:
  - Celestial North Pole
- Major circles:
  - Tropic of Cancer
  - Equator
  - Tropic of Capricorn
- Compass directions:
  - North, East, South, West
- Limb
  - Evenly distributed in 360°





## Declination circles on the rete

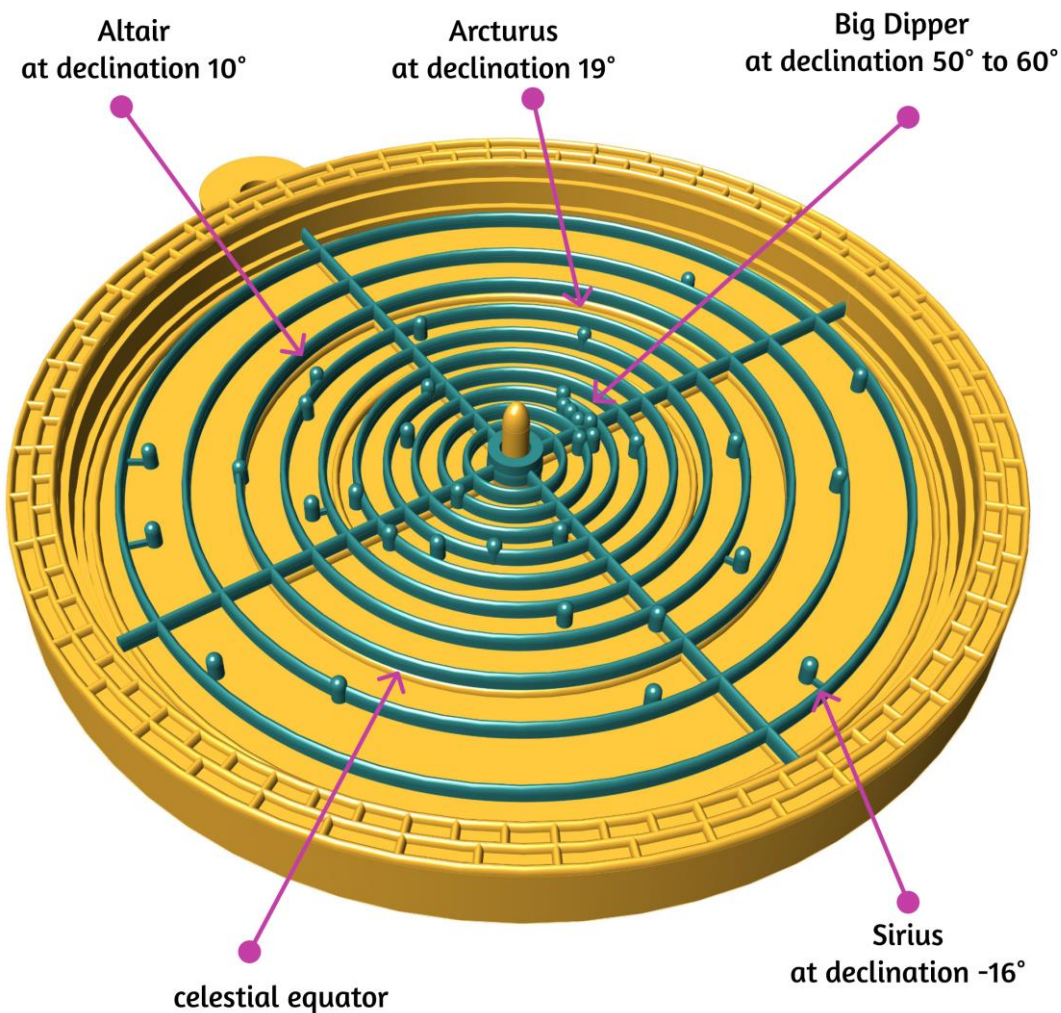
In your 3D printed astrolabe, the declination circles are part of the rete. The ruler also has the same declination scale.

The plate shows the equator, which is at  $0^\circ$  degrees declination. The celestial north pole is at  $90^\circ$  declination. Its projection is the center of the astrolabe.

The tropics are also circles of declination. It is customary to print them on the mater instead of the rete.

Your rete shows the projection of a series of declination circles, starting at  $-20^\circ$ , going up by  $10^\circ$  until  $70^\circ$ . Your rete has two circles below the equator and seven above it.

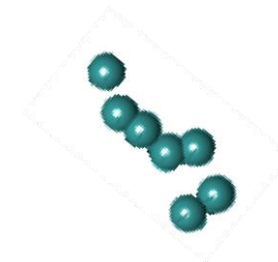
- Plate:
  - Celestial North Pole
  - Equator
  - Tropics
- Rete:
  - Declination below the equator
  - Equator
  - Declination above the equator
- Ruler:
  - declination scale

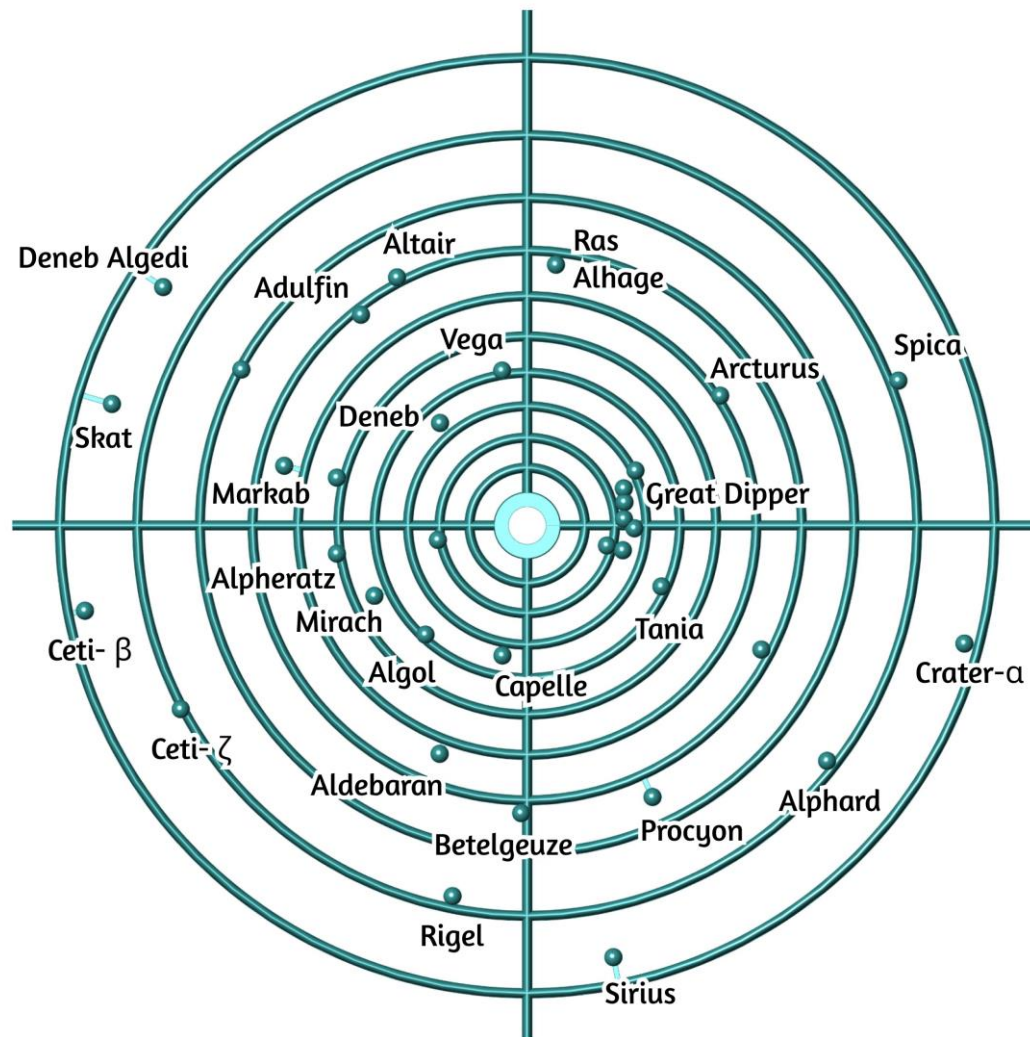


## Stars on the rete

As seen by an observer on planet Earth, stars rotate around the Celestial North Pole. Stars are placed on the rete because the rete can rotate around the Celestial North Pole.

- Celestial equator
- Declination circles
- Stars, among those
  - Altair at 10° declination above the celestial equator
  - Sirius at 16° below the celestial equator
  - Arcturus at 19° above the celestial equator
  - Big Dipper (or Great Bear) at 50° to 60° declination above the celestial equator. The big dipper is composed of seven stars.





## Star chart

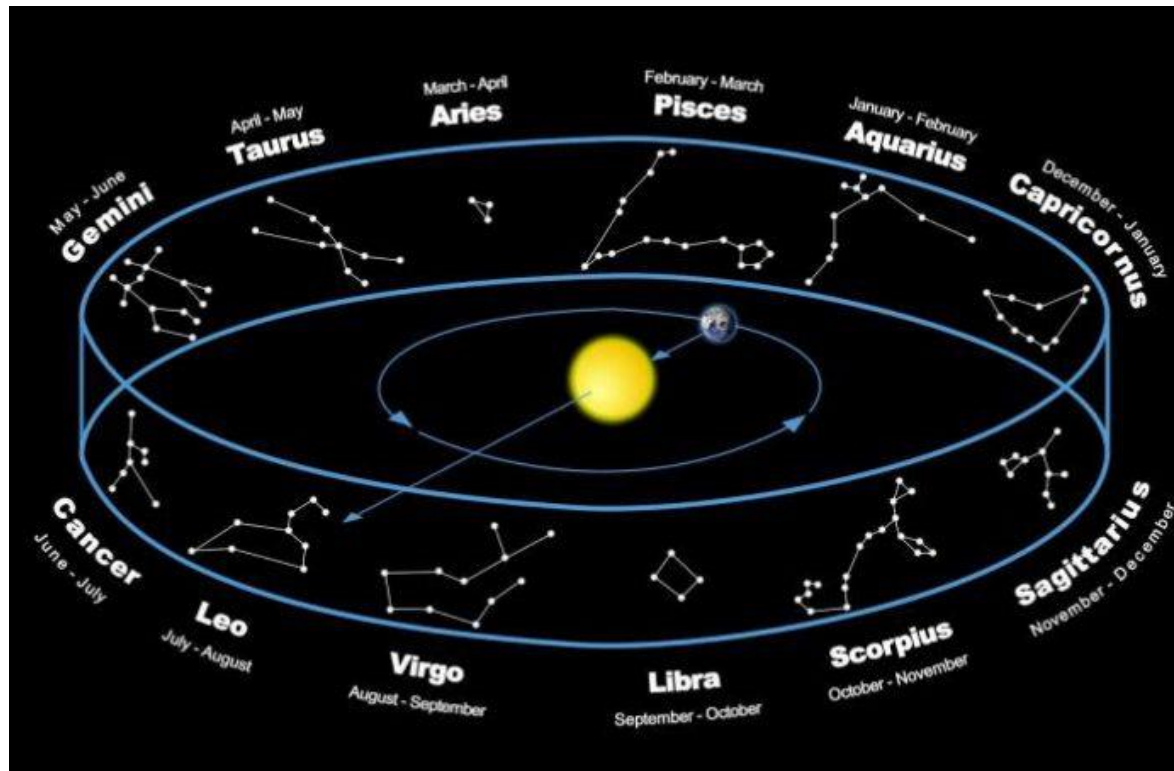
The rete has a star chart depicting the brightest well-known stars.

Due to axial precession (the precession of the equinoxes), the positions of stars change slowly in the ecliptic longitude, but not in the ecliptic latitude. (This is the third coordinate system). A complete precession cycle takes about 26,000 years or  $1^\circ$  every 72 years. As a result, the declination and right ascension of fixed stars change over thousands of years.

Your map shows the positions for 2020.

Printing names on a 3D printed astrolabe was not possible, printing all names on a laminated paper astrolabe was not feasible either. I added the Great Bear as a landmark.





## Ecliptic

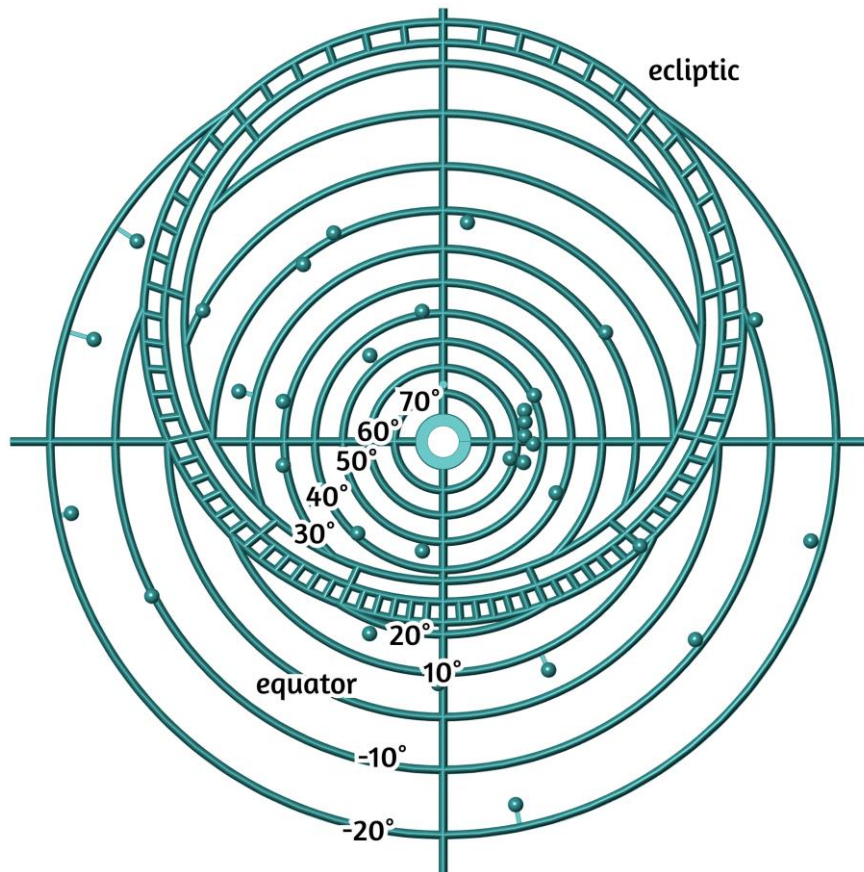
Seen from the planet Earth, the sun makes a tour of the stars every year. Its path is called the ecliptic.

So, the ecliptic is the apparent path of the Sun throughout the course of a year. Because Earth takes one year to orbit the Sun, the apparent position of the Sun takes one year to make a complete circuit of the ecliptic.

With slightly more than 365 days in one year, the Sun moves a little less than  $1^\circ$  eastward every day.

Visibility of a star depends upon the time of the year. To be explained later when dealing with the ecliptic.

For example, the star Altair is in the direction of Capricorn. In winter, when the sun is in Capricorn, you cannot observe Altair during the day. Therefore, Altair is best visible from May to September.

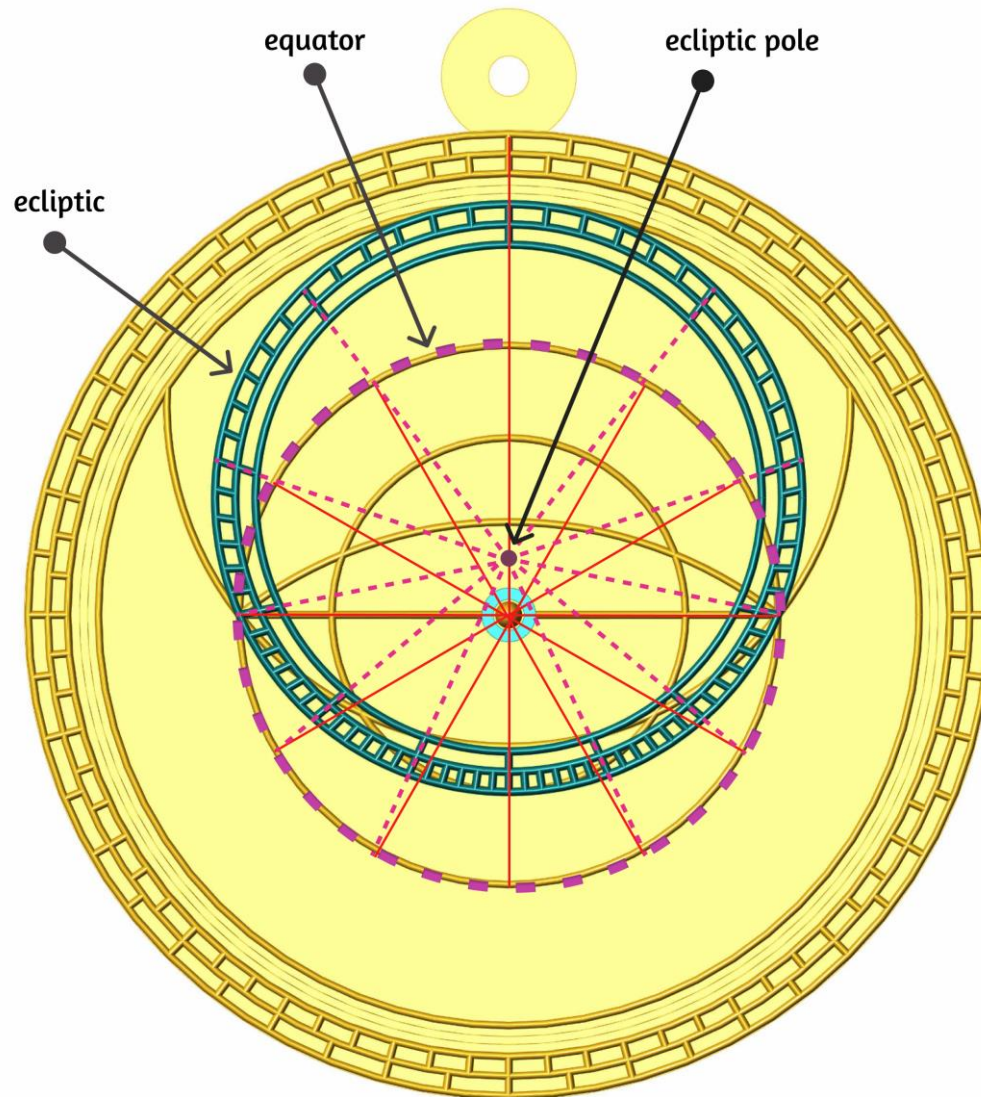


## Rete with ecliptic and stars

The rete rotates across the plate to mimic the daily movement of the sun and the stars around the North Pole. The ecliptic shows the position of the Sun during the year. The sun moves between  $+23.4^\circ$  (Tropic of Cancer) and  $-23.4^\circ$  (Tropic of Capricorn).

Throughout the year, the sun moves between the Tropic of Cancer and the Tropic of Capricorn and passes the celestial equator twice a year.

- Declination circles
  - Seven declination circles above equator:  $10^\circ$ ,  $20^\circ$ , etc to  $70^\circ$
  - The equator is the  $0^\circ$  circle
  - Two declination circles are below the equator:  $-10^\circ$ ,  $-20^\circ$
- Ecliptic
  - The ecliptic is a circle
  - The ecliptic is the outer circle.
  - The position of the Sun is at the outer circle
  - The inward circles has the division in twelve zodiac signs and a subdivision in degrees



## Division of the ecliptic

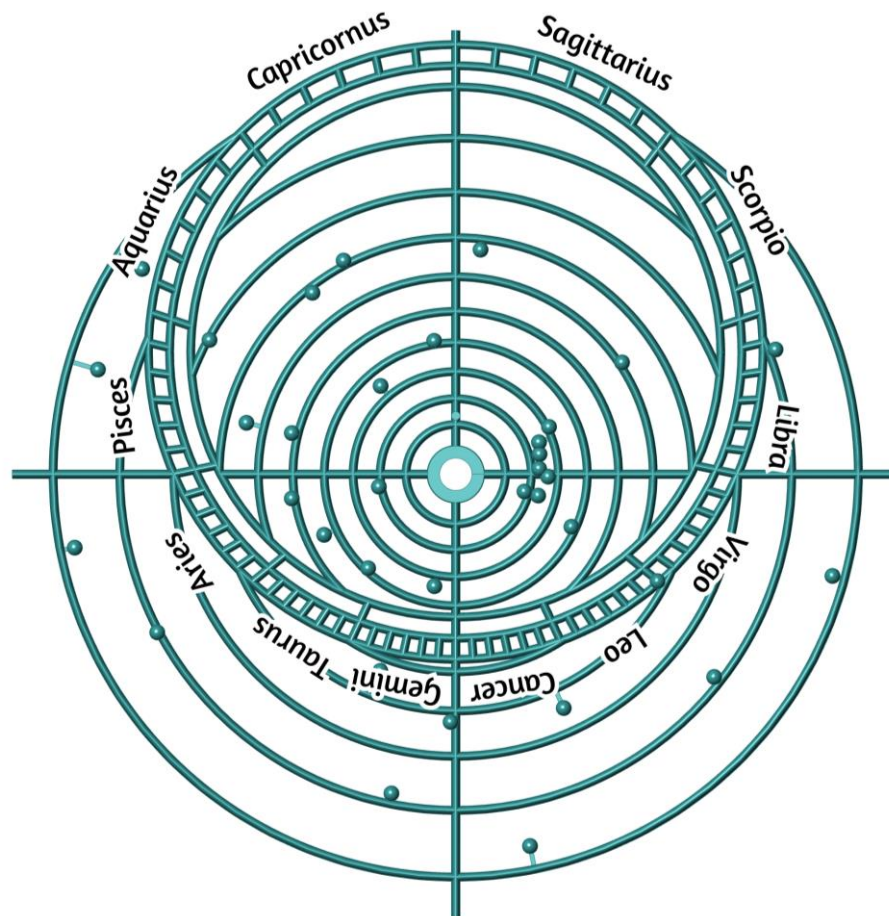
The sun travels through the ecliptic in a year. In contrast to the equator with its periods of equal length, the ecliptic is divided into twelve periods of unequal length. Centered around the celestial north pole, every arc of the equator is exactly  $30^\circ$ .

All lines dividing the ecliptic start in the ecliptic pole and intersect in the corresponding point at the equator.

The ecliptic pole is not the center of the ecliptic circle, but the stereographic projection of the pole of the ecliptic plane. So, the arcs are not centered around the ecliptic pole, some arcs are less than  $30^\circ$  while others are greater than  $30^\circ$ .

- Equator
  - Divided into twelve periods of  $30^\circ$
- Ecliptic
  - Center is halfway both tropics
  - Ecliptic pole is base for the rays that divide the ecliptic.
  - Divided into twelve periods
  - Each period is subdivided into six smaller periods of  $5^\circ$

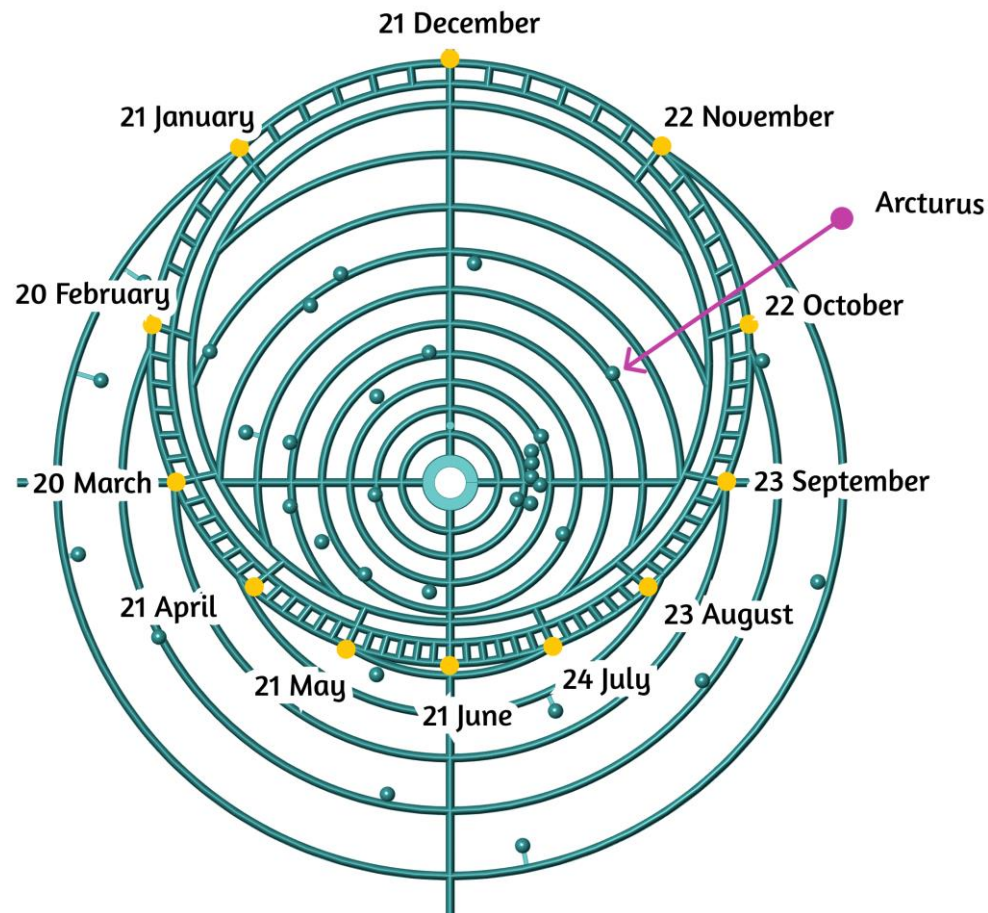




## Zodiac signs

Throughout the year, the sun moves counterclockwise along the ecliptic.

- The ecliptic is divided into twelve zodiacal signs:
  - Sagittarius
  - Scorpio
  - Libra
  - Virgo
  - Leo
  - Cancer
  - Gemini
  - Taurus
  - Aries
  - Pisces
  - Aquarius
  - Capricornus
- Horizontal and vertical lines
  - Horizontal line is from 0° Aries to 0° Libra
  - Vertical line is from 0° Cancer to 0° Capricorn



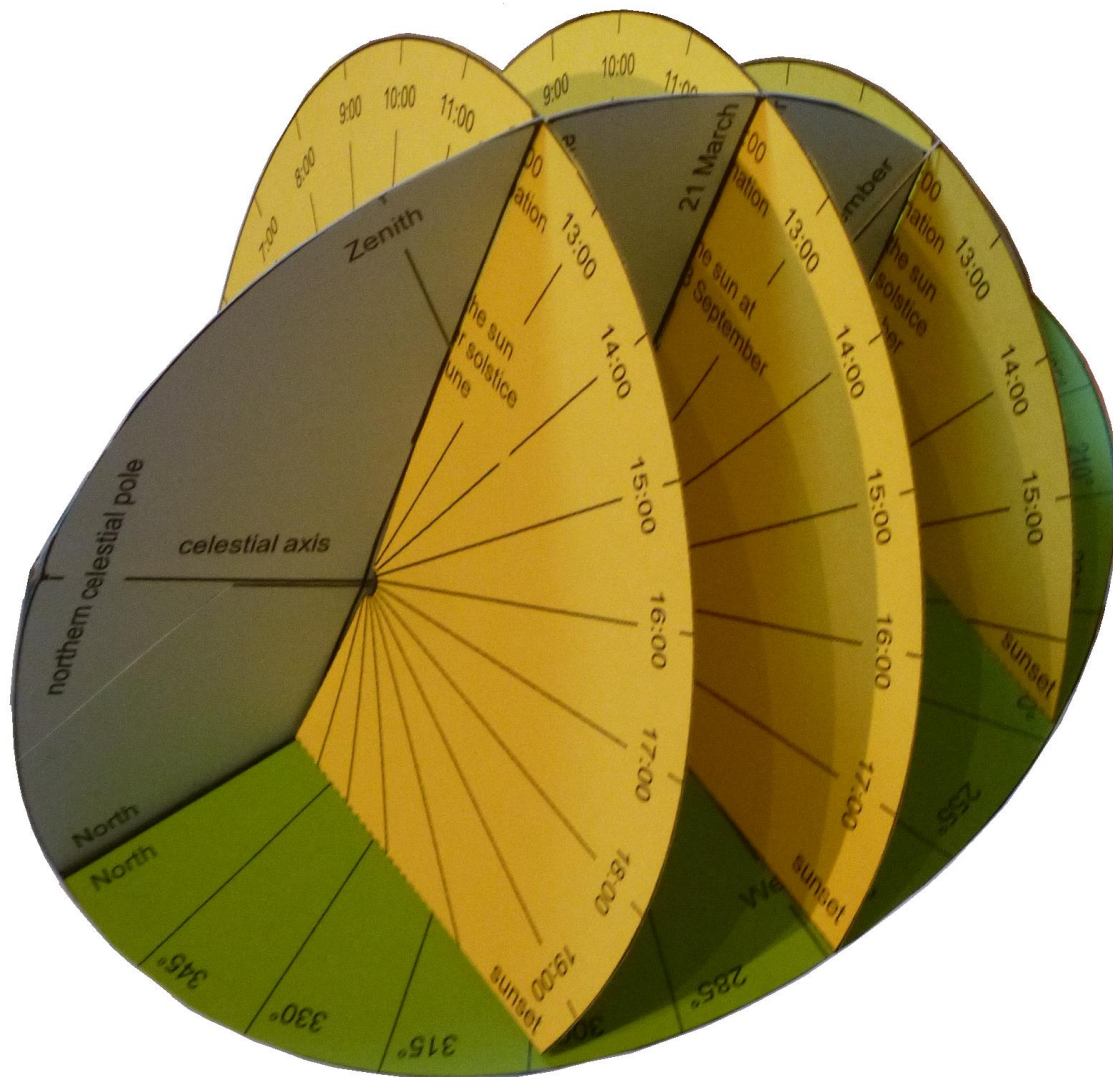
## Rete with calendar dates

You may rotate the rete across the plate. During the year, the sun moves counterclockwise along the ecliptic, which is divided into the twelve signs of the zodiac.

In late November, the sun is at the beginning of zodiac sign Sagittarius, but due to precession, the zodiac is one full position ahead of the constellations in the sky. In late November, the sun is in the direction of the stars of the constellation Scorpio.

Similarly, at the end of October, the sun is in the direction of Arcturus. According to the zodiac, that is the sign of Scorpio, but in reality, the stars of the constellation Libra are there.

- Declination circles  
from  $-20^\circ$ , to  $-10^\circ$ , to  $-0^\circ$ , to  $+10^\circ$ , etc..
- Ecliptic
  - The position of the Sun throughout the year
  - The ecliptic crosses the meridian line (North-South) at the tropics: declination  $+23,4^\circ$  and declination  $-23,4^\circ$ .



## Planes

Your paper model explains that there are multiple planes to consider.

For every latitude, one should have a different model. The celestial axis is equivalent to the earth rotation axis.

The angle between the celestial axis and the horizon plane is equal to the latitude.

A town at the earth equator has latitude  $0^\circ$ . Here the celestial axis is parallel to the horizon plane.

At the North Pole, the latitude is  $90^\circ$ . The celestial axis is perpendicular to the horizon plane.

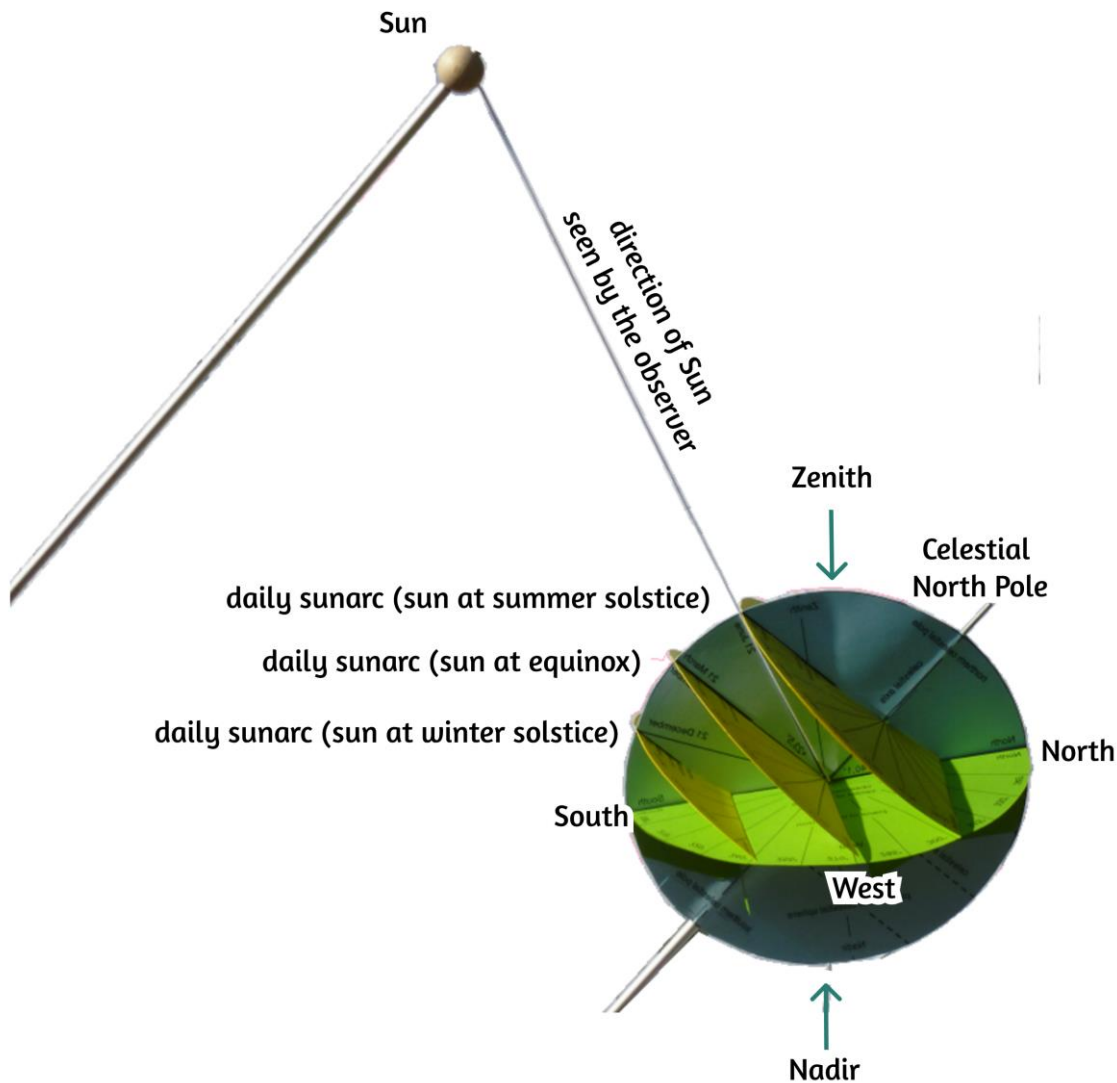
- Horizon plane (green)
  - Compass directions
  - Azimuth
- Meridian plane (blue)
  - Celestial axis
  - Latitude
- Daily sunarc planes (yellow)
  - Plane with Tropic of Cancer
  - Equatorial plane
  - Plane with Tropic of Capricorn

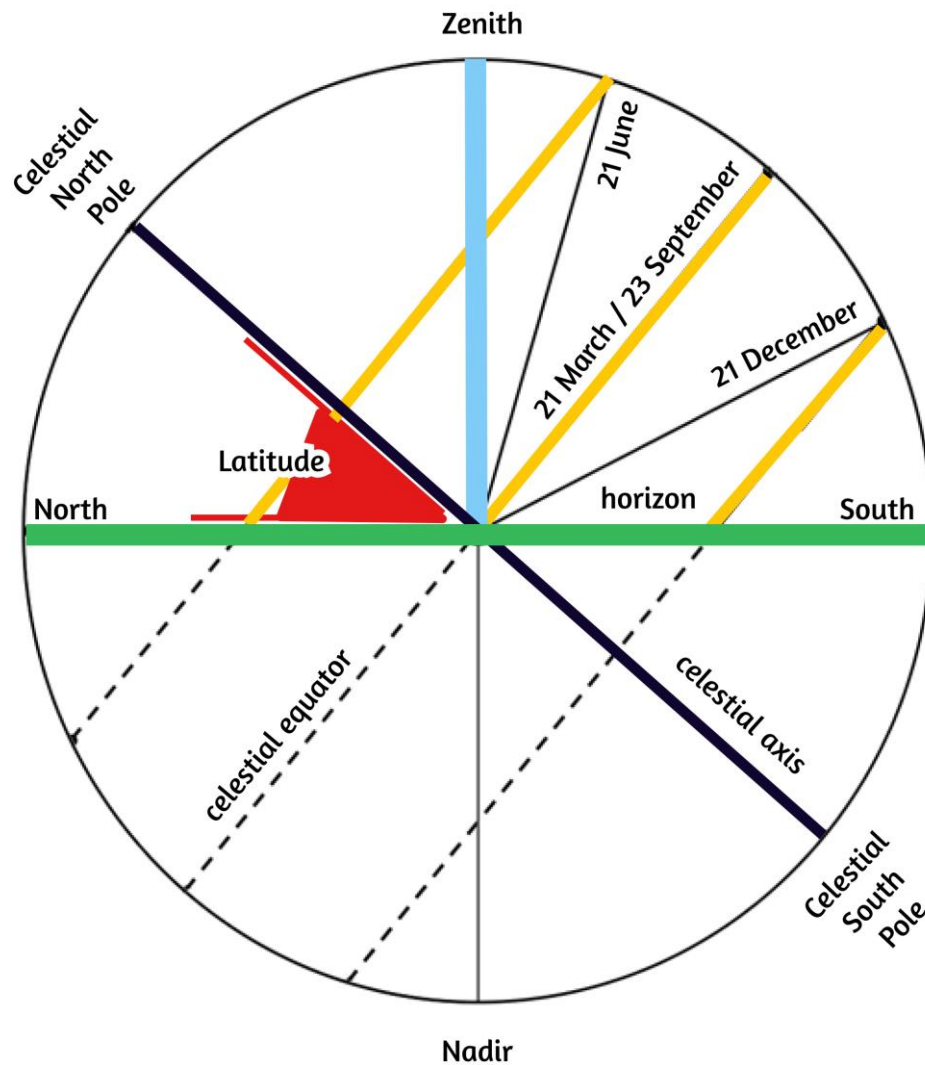


## Directions

Say all the stars are stuck on a sphere.  
Within the sphere there is:

- An observer at the center of the horizon plane (the same as the center of the meridian plane)
- a horizon plane (green) with four compass directions:
  - North, East, South, West
- a meridian plane (blue)
  - Zenith at the top
  - Nadir below
  - Celestial axis (same as earth's axis of rotation)
- Daily sunarc planes (yellow) perpendicular to the celestial axis
  - Sun at summer solstice means that Sun is in the Tropic of Cancer
  - Sun at equinox means that Sun is in the celestial equator
  - Sun at winter solstice means that Sun is in the Tropic of Capricorn





## Meridian plane

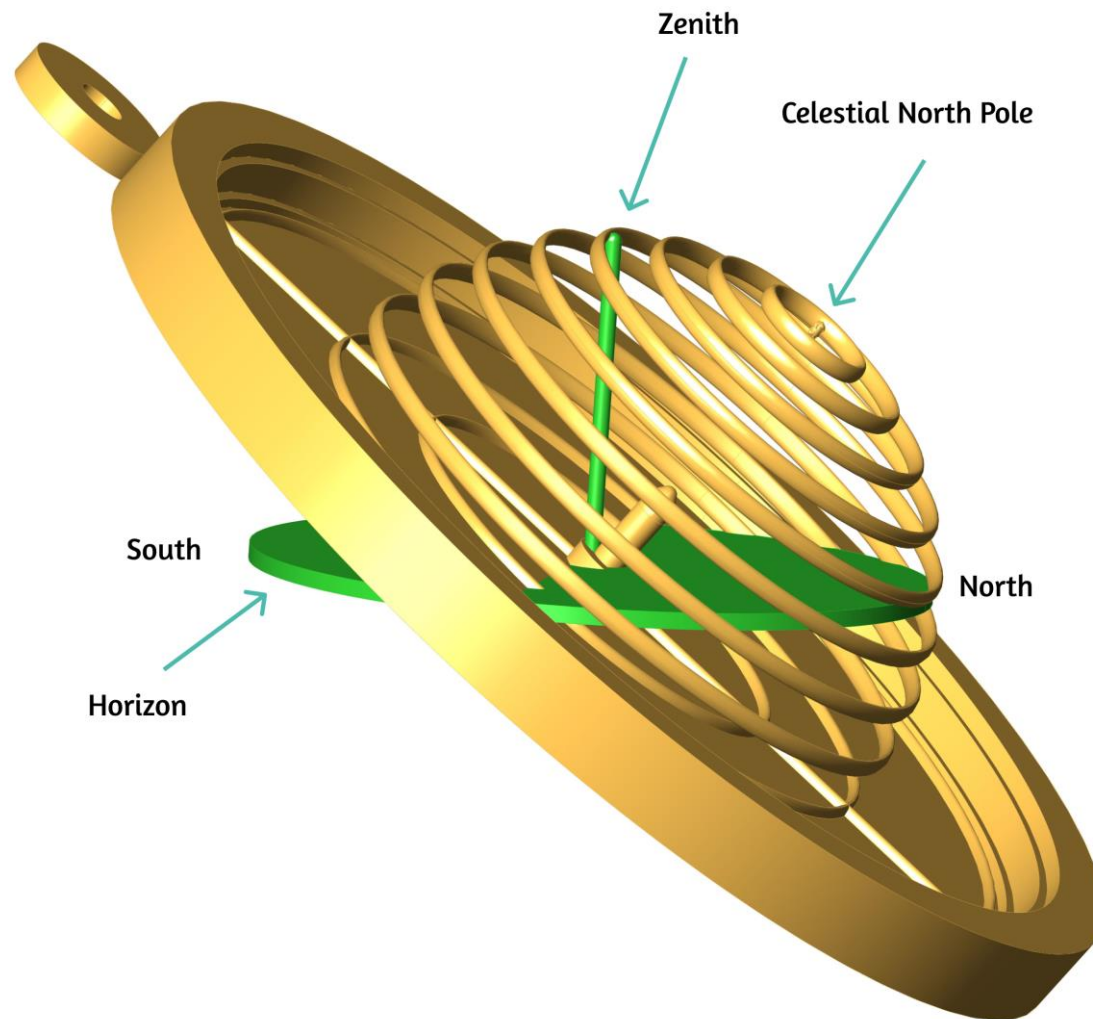
The meridian plane is perpendicular to the horizon plane. Zenith is on top and Nadir is below.

Latitude is the angle between the horizon and the celestial axis.

The angle between the celestial axis and Zenith is called colatitude, the complement of latitude.

All daily sunarcs are equidistant. The angle between a daily sunarc (yellow) and the horizon is the colatitude.

At noon during the equinox, the angle between the sun and the horizon plane is colatitude. At noon during the summer solstice, this angle is  $23,4^\circ$  more than colatitude. At noon during the winter solstice, this angle is  $23,4^\circ$  less than colatitude.



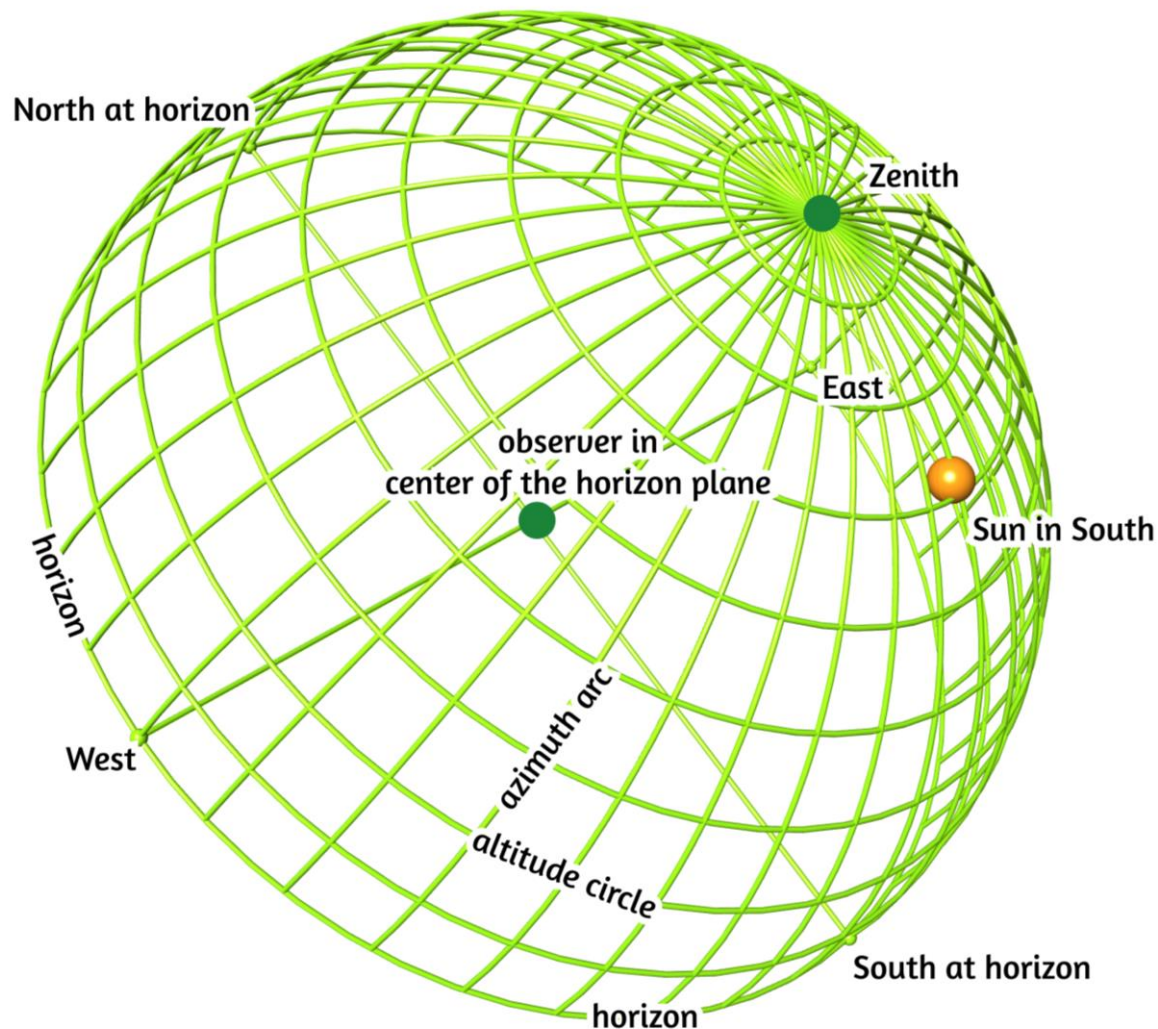
## Horizon plane and Zenith

The horizon plane makes an angle with the equatorial plane of the astrolabe.

The angle is equal to the colatitude

- On the equator, latitude is  $0^\circ$  and colatitude is  $90^\circ$ .
- On the North Pole, latitude is  $90^\circ$  and colatitude is  $0^\circ$ .
- In Amsterdam, latitude is  $52^\circ$  and colatitude is  $38^\circ$ .
- In Istanbul, latitude is  $41^\circ$  and colatitude is  $49^\circ$ .



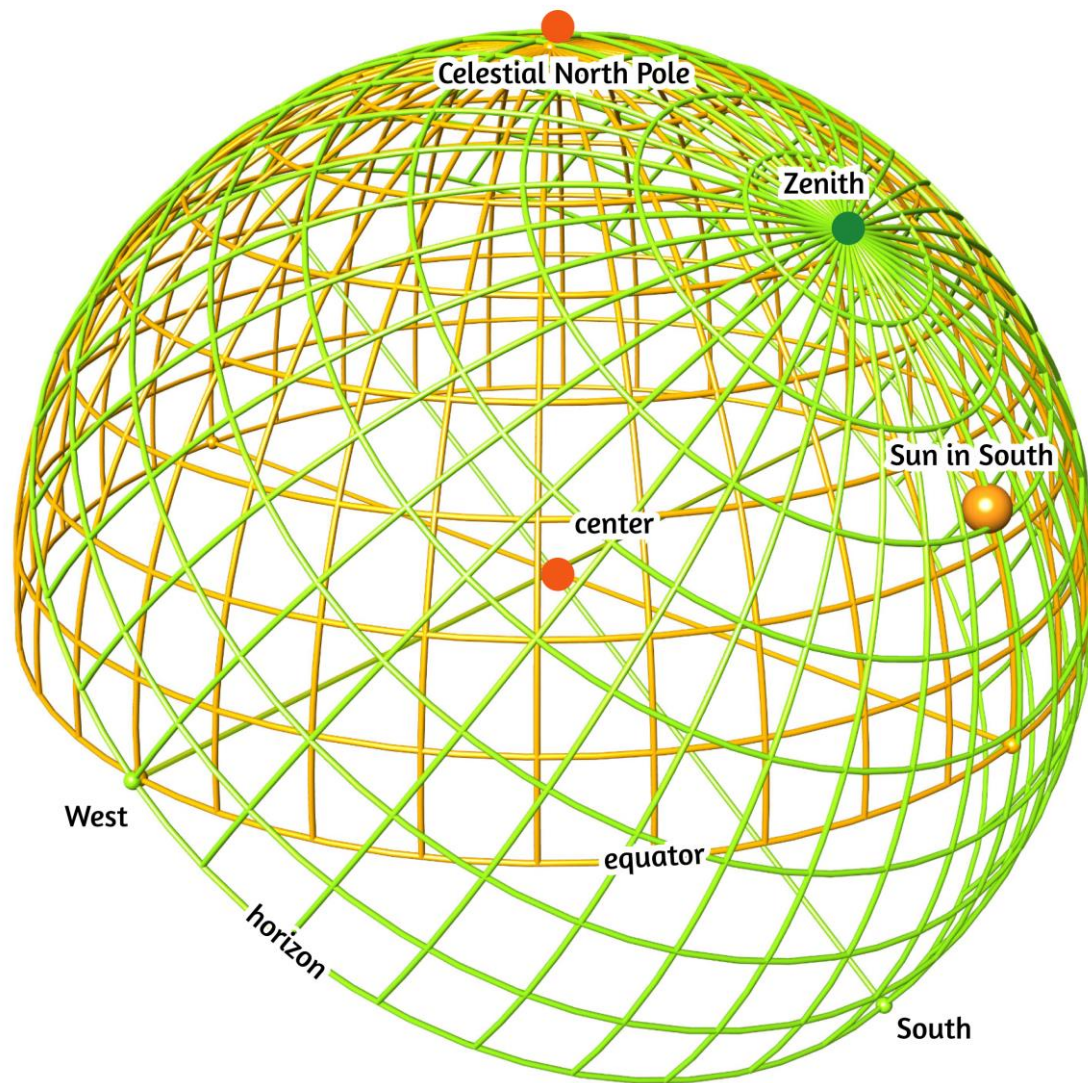


## Horizon sphere

Imagine a spherical dome over the circular horizon plane. All celestial bodies move over this sphere. Objects rise and set. For example, the sunrises in the morning and sets in the evening. Likewise, the stars rise and set.

This sphere is also like a map. The grid lines are the altitude circles and the azimuth arcs. At a specific time, each celestial object has a position, but a few minutes later, due to the earth rotation, every object has a new position.

In this illustration, the horizon is tilted in accordance with latitude, reflecting a horizontal equator.

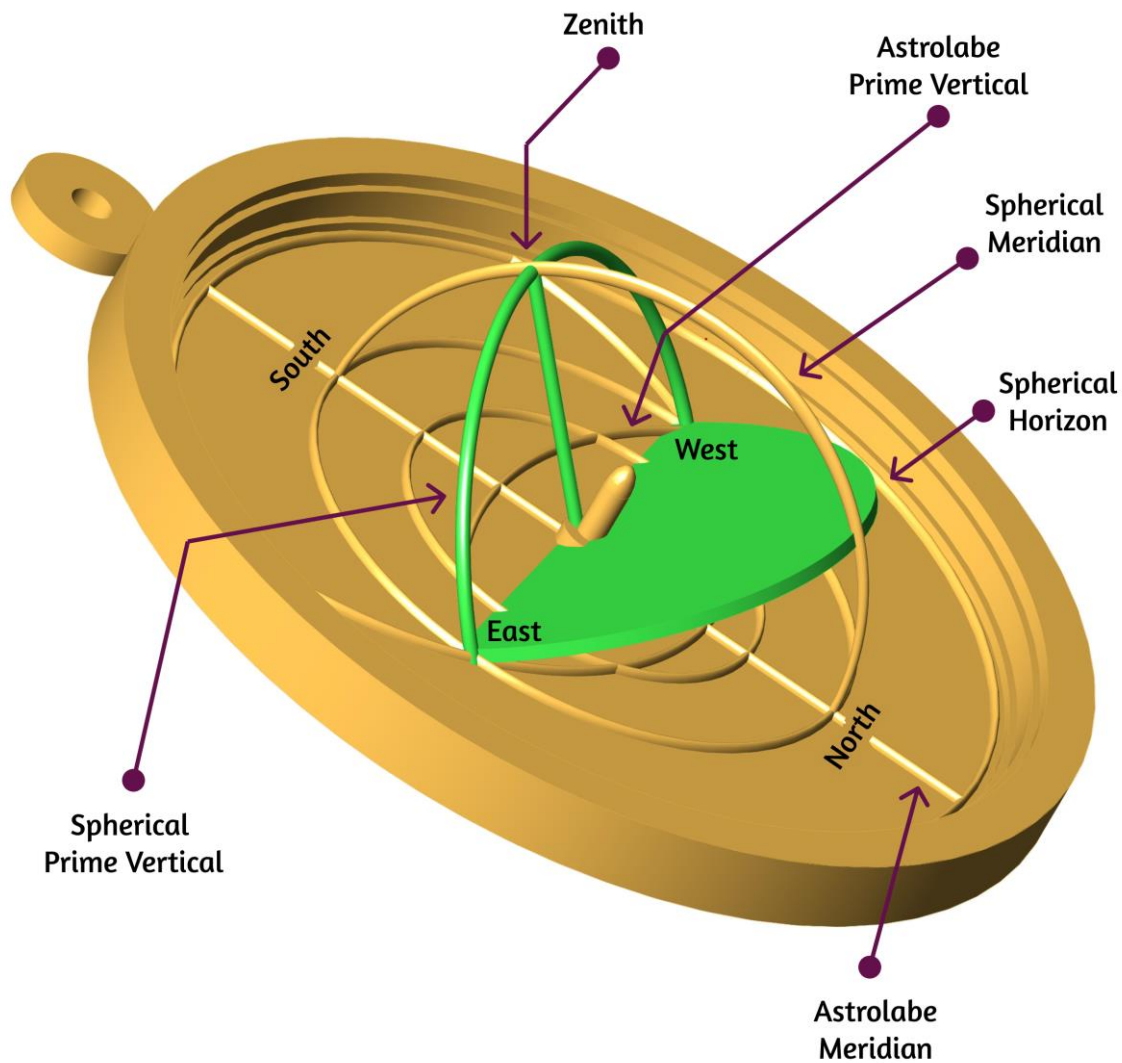


## Celestial sphere and horizon sphere

Here, we put the northern hemisphere (of the celestial sphere) horizontal. The horizon sphere is tilted according to the latitude.

This illustration shows the sun in the south at noon during the summer solstice. The sun is just  $23.4^\circ$  above the equator. The angle between the horizon and the equator is the colatitude. Suppose the latitude for Amsterdam is  $52.4^\circ$ . So the colatitude is  $37.6^\circ$ . So the sun's altitude is  $61.0^\circ$ .

In winter, when the sun is below the equator, you can see celestial bodies on the horizon in Amsterdam with a declination of  $-37.6^\circ$  at most. The sun is at  $-23.4^\circ$ , which means the sun is visible at noon at  $14.2^\circ$  altitude during the winter solstice in late December.



## Prime Vertical

Prime vertical is the arc from East to Zenith to West. It is an azimuthal arc, perpendicular to the meridian from North to East and perpendicular to the horizon plane.

The projection of the prime vertical is an arc on the astrolabe plate. The projection is definitely not the right horizon (from East to center to West) because the right horizon is the projection of the arc from East to the Celestial North Pole to West.

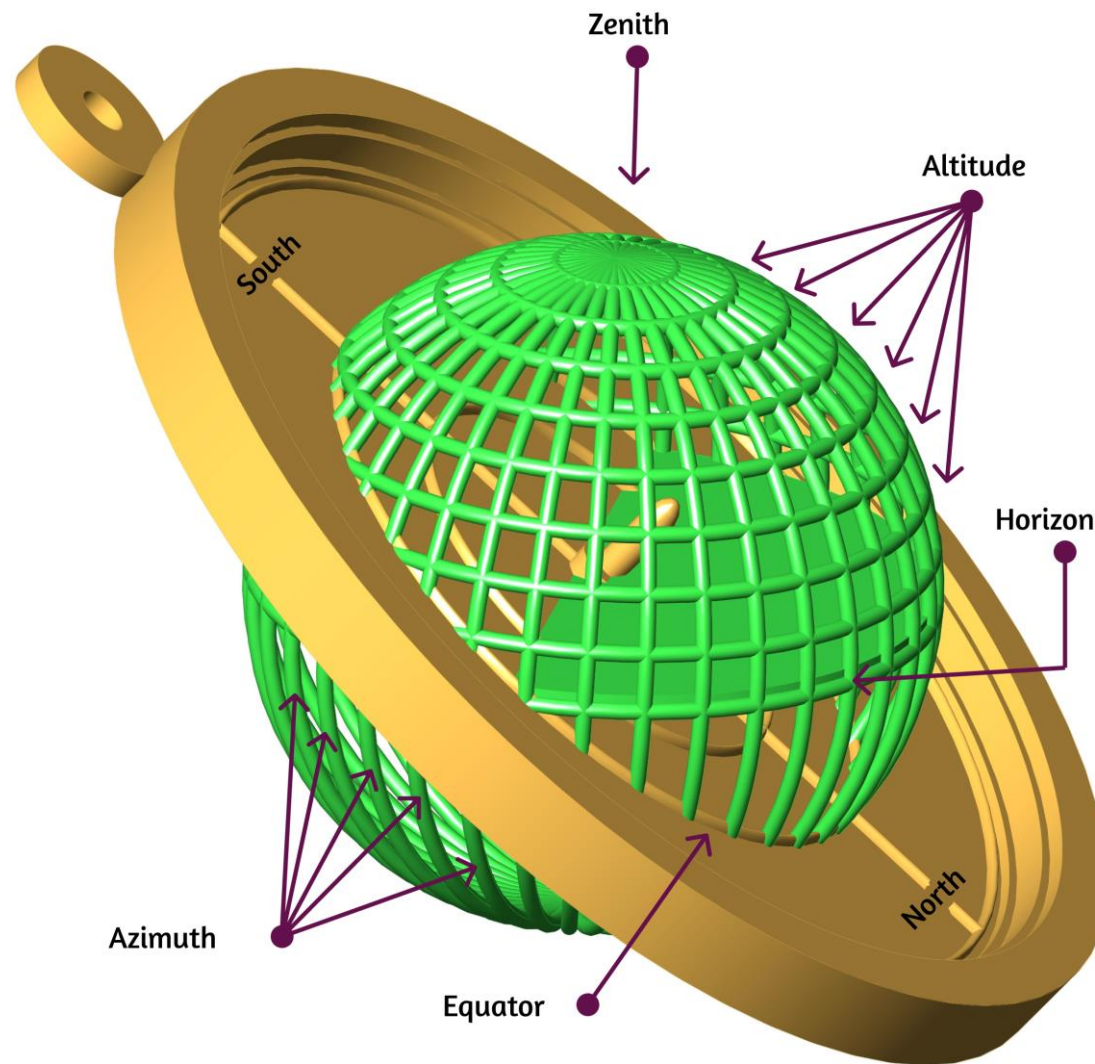
- Horizon
- Zenith
- Prime Vertical



## Altitude and Azimuth

The coordinate system of the horizon plane is composed of direction relative to north-south axis and height above the horizon.

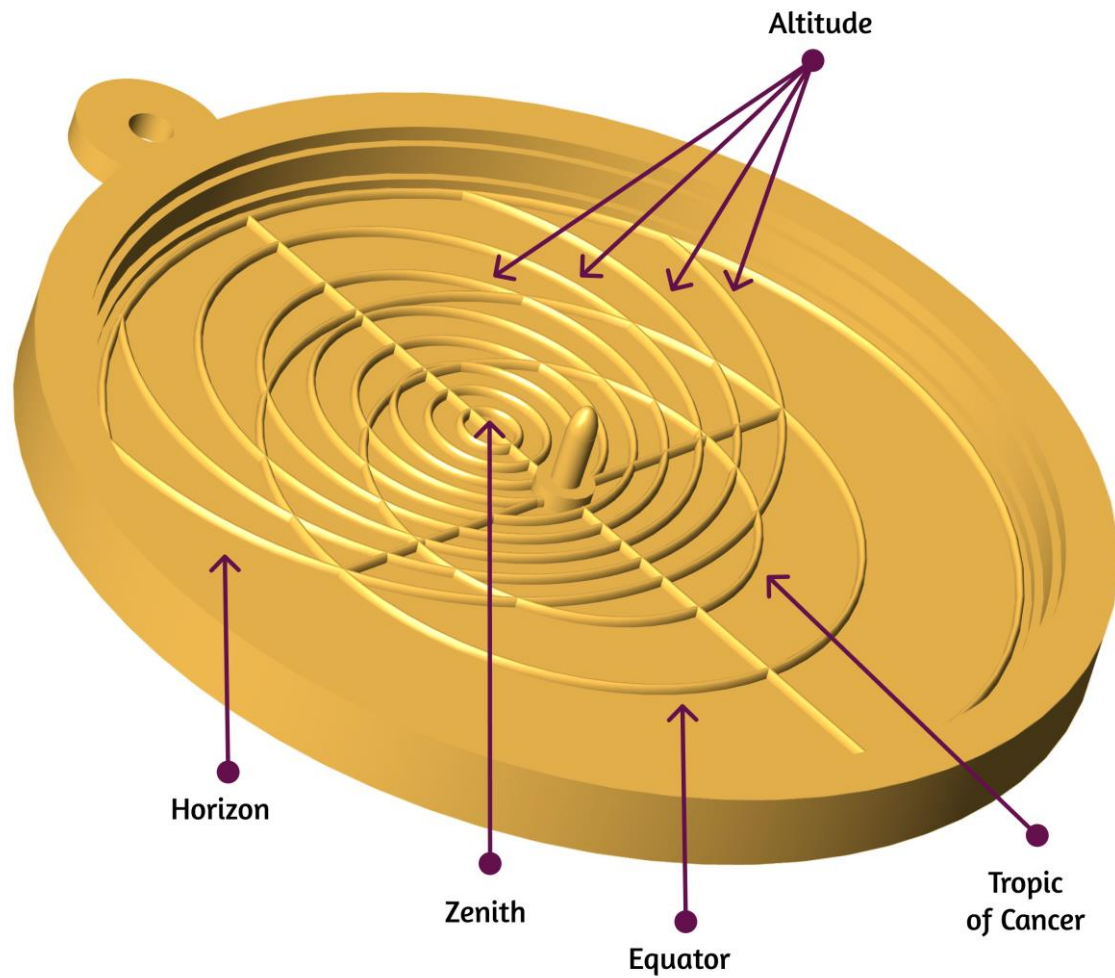
- Azimuth is direction relative from South
- Altitude is height above horizon.
- Altitude arcs are called almucantars

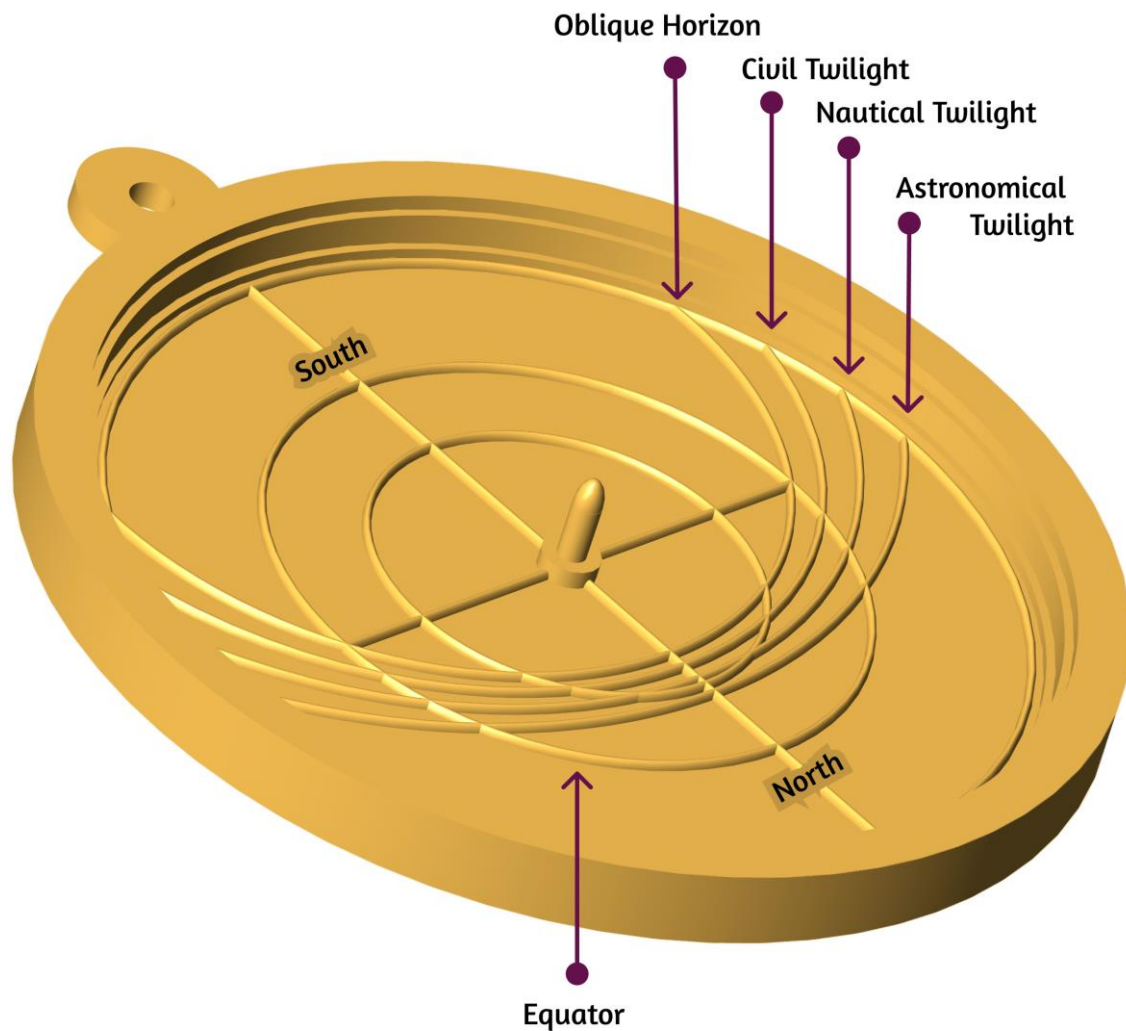


## Projection of horizon and altitude

The horizon is a large circle passing East and West. Zenith is not its center.

Altitude is height above horizon. Altitude circles are not concentric, although all their centers are on the meridian line





## Projection of Twilight

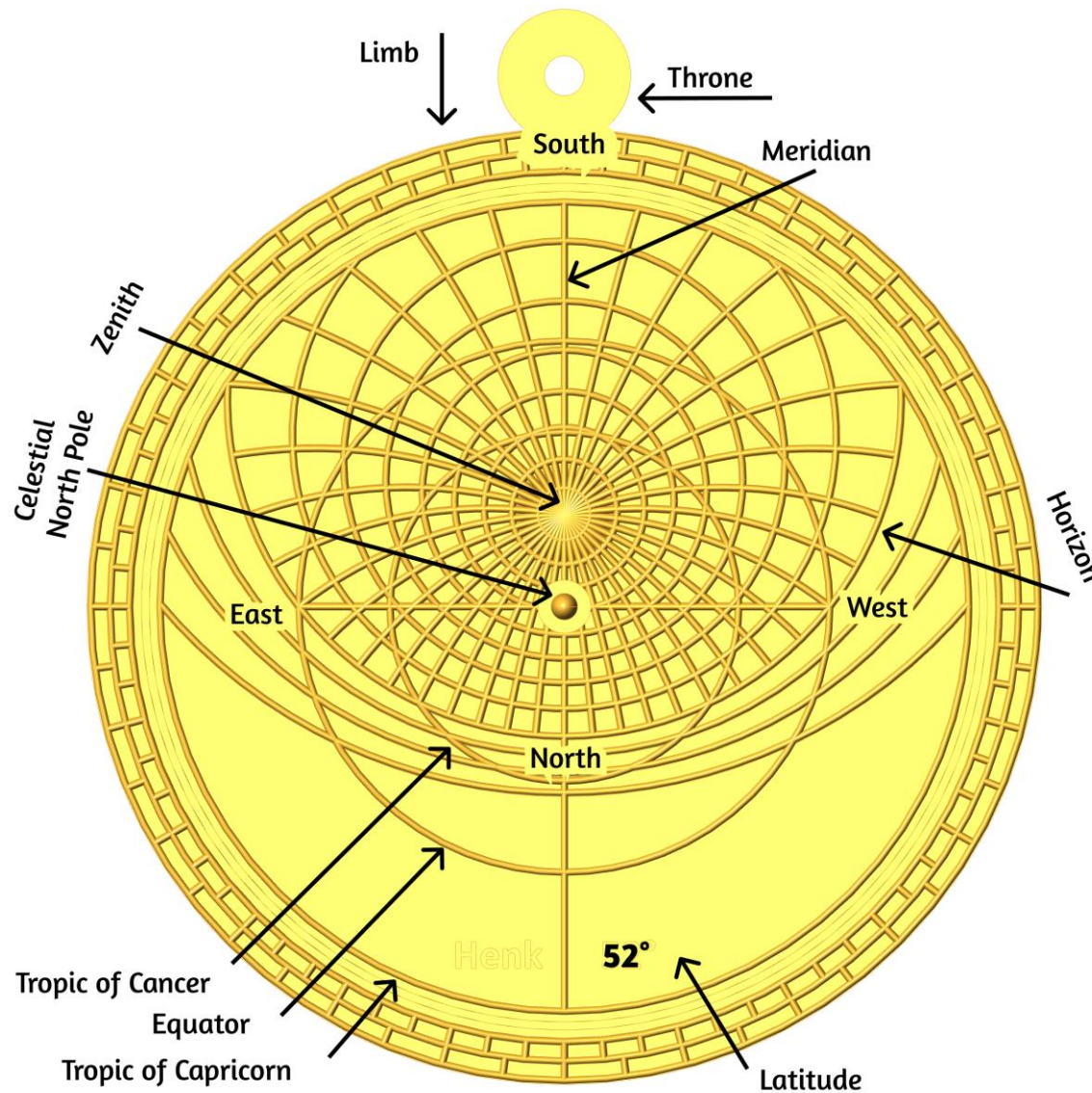
Even when the sun is below the horizon, the sky is lit because the atmosphere. We call it twilight. There are three kinds of twilight:

- Civil twilight until the sun is  $6^\circ$  under the horizon. During this period, artificial illumination is not necessary
- Nautical twilight until the sun is  $12^\circ$  under the horizon. At sea, one can not see the horizon anymore.
- Astronomical twilight until the sun is  $18^\circ$  under the horizon. There is no more sunlight at all. The light of the stars is no longer irradiated by the light of the sun

Note that

- the size of the sun is  $0,5^\circ$
- the atmosphere bends sunlight
- particles in the atmosphere cause refraction

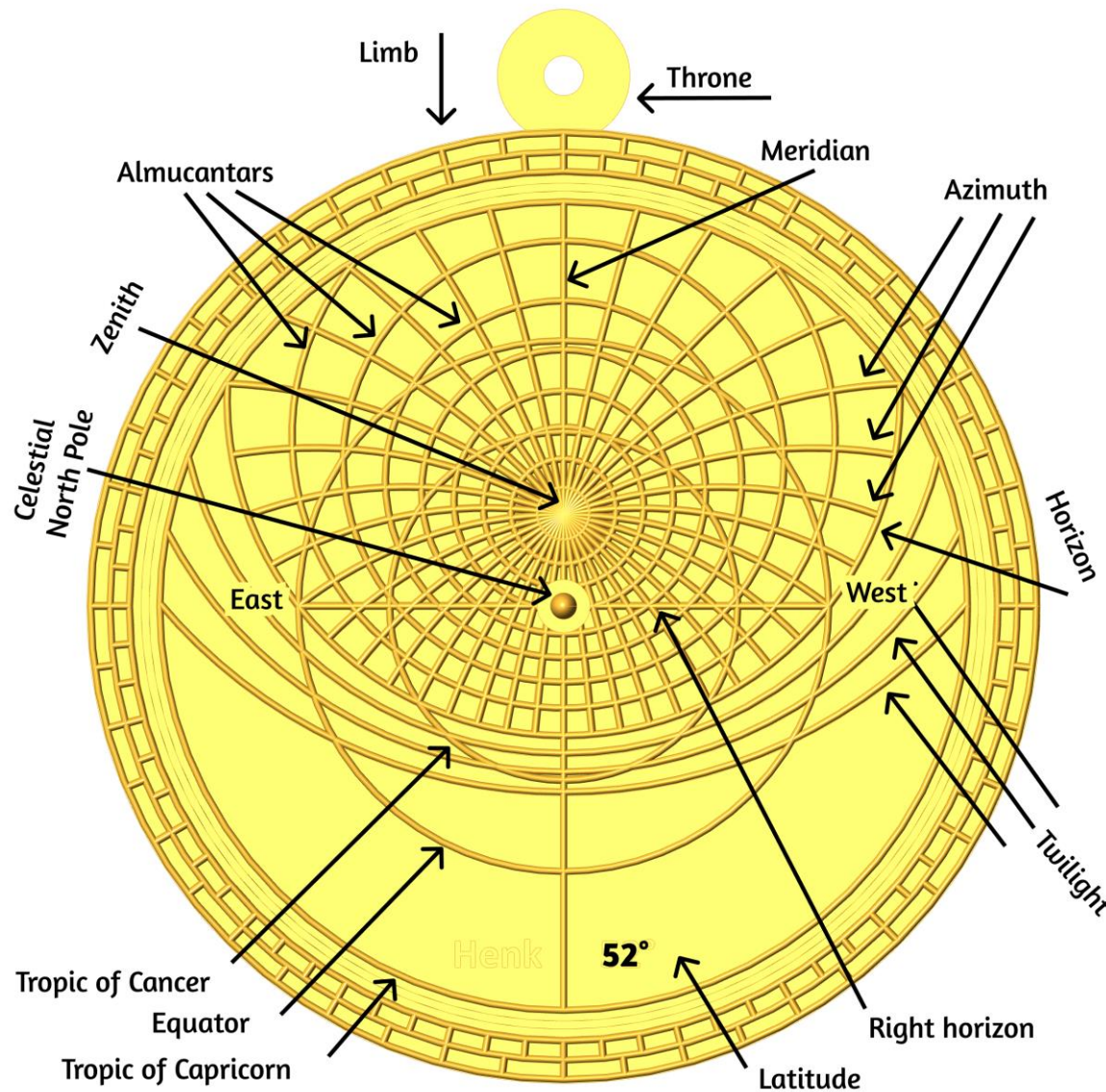




## Front side with local horizon

Keep your astrolabe flat like a compass and point the throne southward for a natural orientation of the movement of the Sun along the sky.

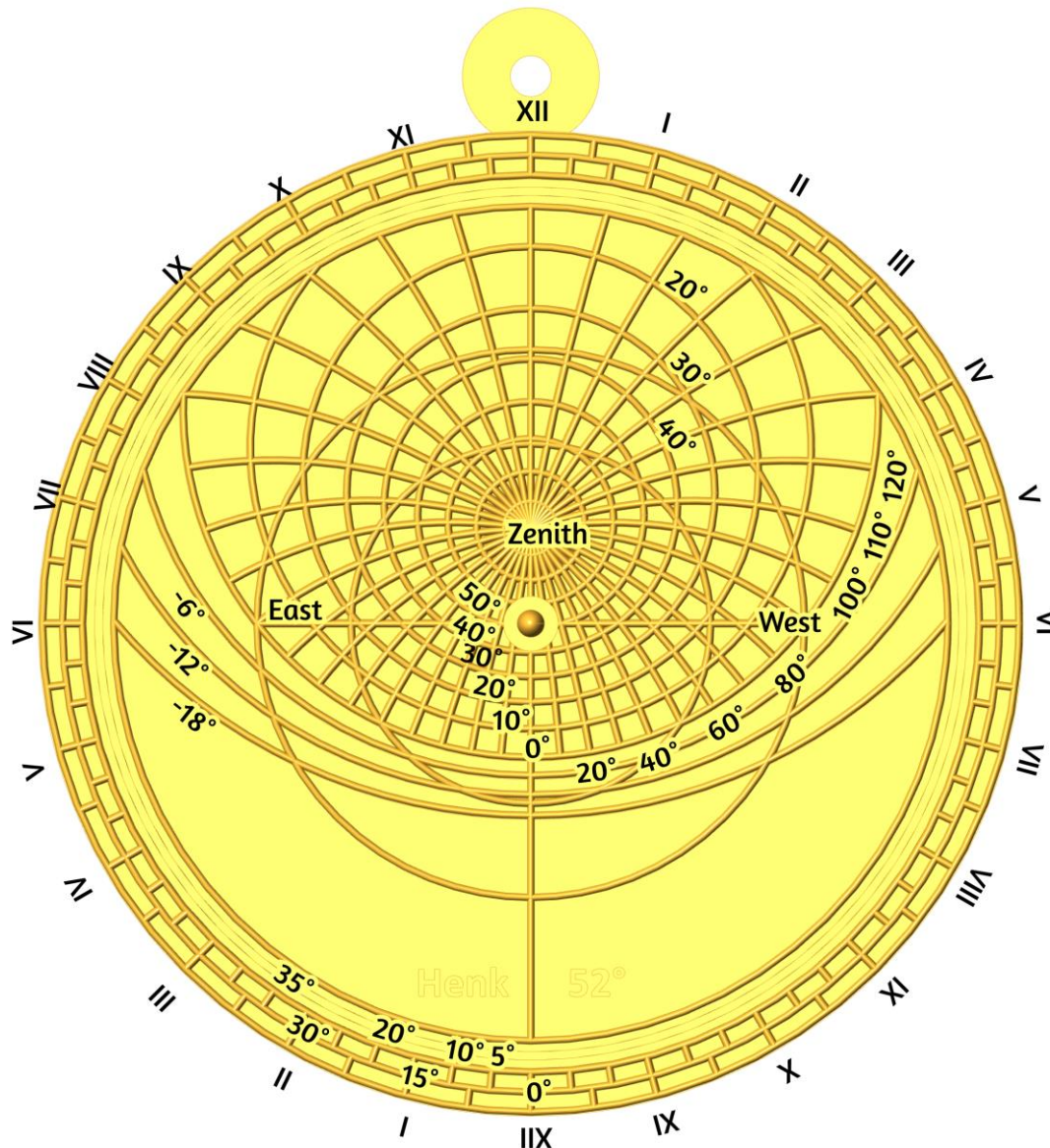
- The limb is the outer ring, divided into
  - 24 hours
  - 48 half hours
  - 72 portions of 5°
- Three major circles
  - Tropic of Capricorn
  - Equator
  - Tropic of Cancer
- Four special points:
  - Celestial North Pole
  - Zenith, the point in the heavens right above your head
  - East and West are points on the equator and the horizon
- The horizon at your latitude is an arc with the directions East, North and West.
- The meridian is a vertical line through four points
  - South at horizon
  - Zenith
  - Celestial North Pole
  - North at horizon



## Front side with local grid

- Azimuth arcs divide the horizon into equal portions
  - Azimuth arcs are part of circles, from the horizon, through Zenith, back to the horizon at an opposite point on the horizon
  - 36 arcs have been printed: 0°, 10°, 20°, etc..
  - The arc from East to Zenith to West is called the Prime Vertical
  - The arc from South to North is called the meridian
- Almucantars are circles of equal altitude above the horizon
  - The horizon is the circle of 0° altitude
  - Seven arcs (circles) are printed around Zenith
- Three twilight arcs are below the horizon
- The right (equatorial) horizon is a horizontal line through the center of the astrolabe from East to West



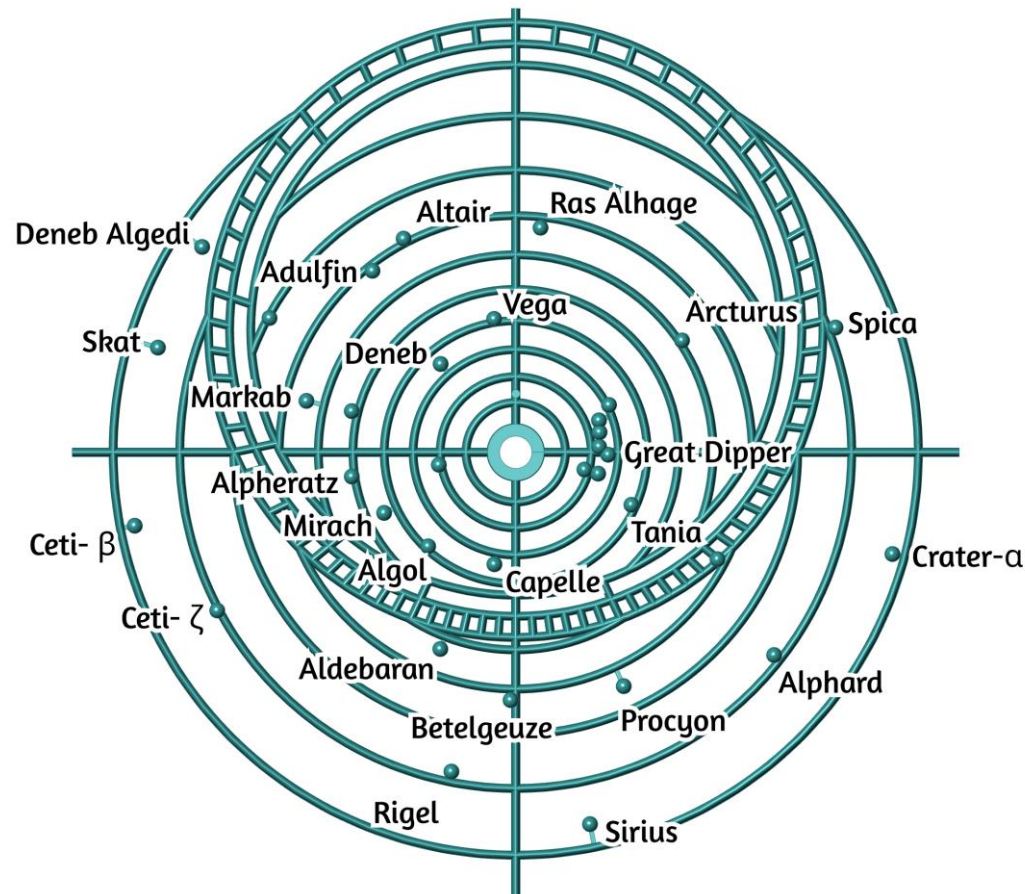


## Front side with grid in degrees

Almucantars and azimuth arcs divide the sphere above the horizon into spherical quadrangles. The resulting grid is expressed in degrees.

- The division of the limb
  - 24 hours,
  - counting twice from I to XII
  - 360 degrees
- Almucantars are circles of equal altitude above the horizon
  - 0° altitude is the horizon
  - 10°, 20°, ..., etc until 70°
  - The 80° circle is not printed
  - 90° altitude is Zenith
- Azimuth arcs from horizon to Zenith
  - 0°, 10°, 20°, ..., until 360°
- Twilight
  - Civil twilight at -6°
  - Nautical twilight at -12°
  - Astronomical twilight at -18°





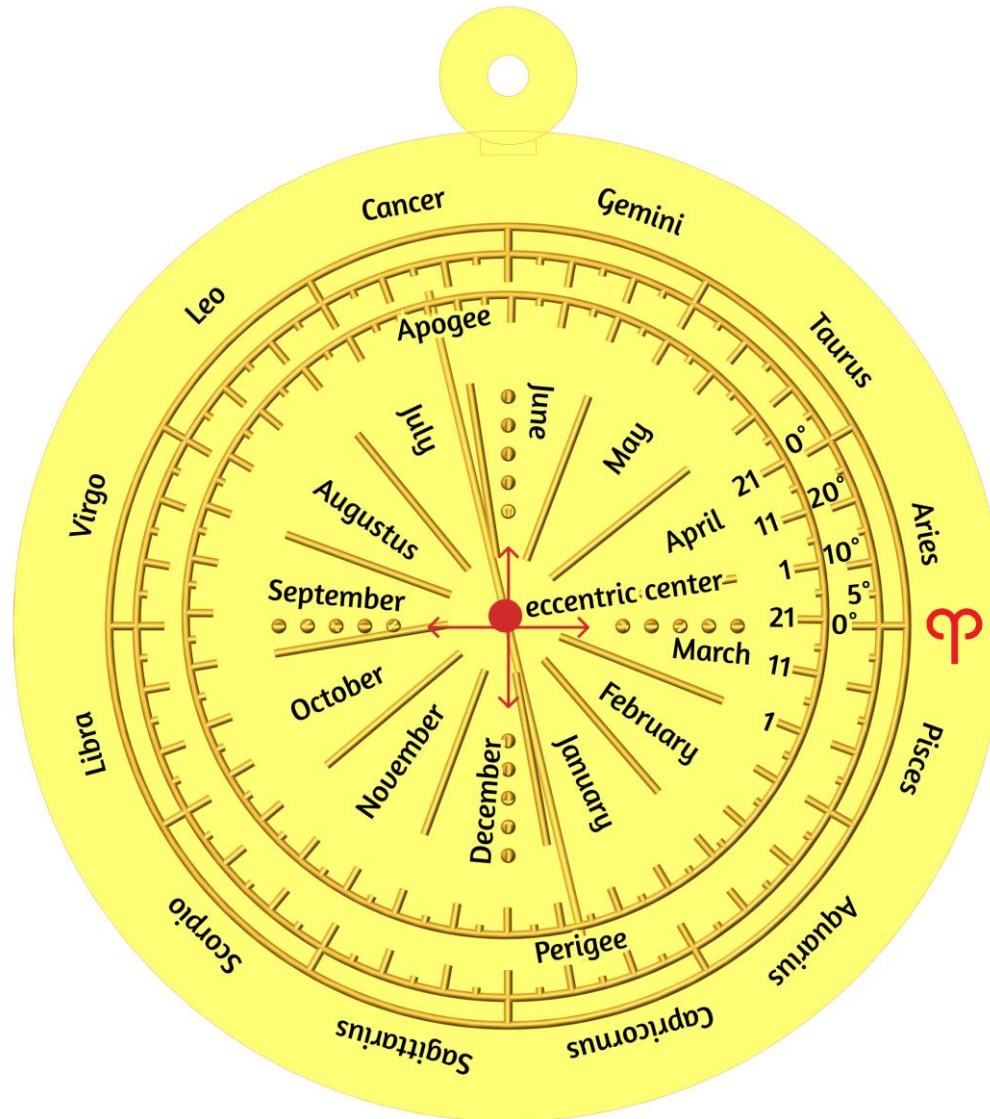
## Spider with star map

The spider rotates across the plate. The spider has a star map and the plate reflects the sphere above the local horizon.

- Ecliptic
  - The position of the Sun is at the outer circle
  - Division into twelve zodiac signs
- Declination circles
  - Two declination circles below the equator ( $-20^\circ$  and  $-10^\circ$ )
  - equator ( $0^\circ$ )
  - seven circles above the equator ( $10^\circ$ ,  $20^\circ$ , ...,  $70^\circ$ )
- Stars
  - The dots mark the most bright stars
  - The Pole Star is at the center of the spider.

The Great Dipper or Big Dipper is made of seven stars:

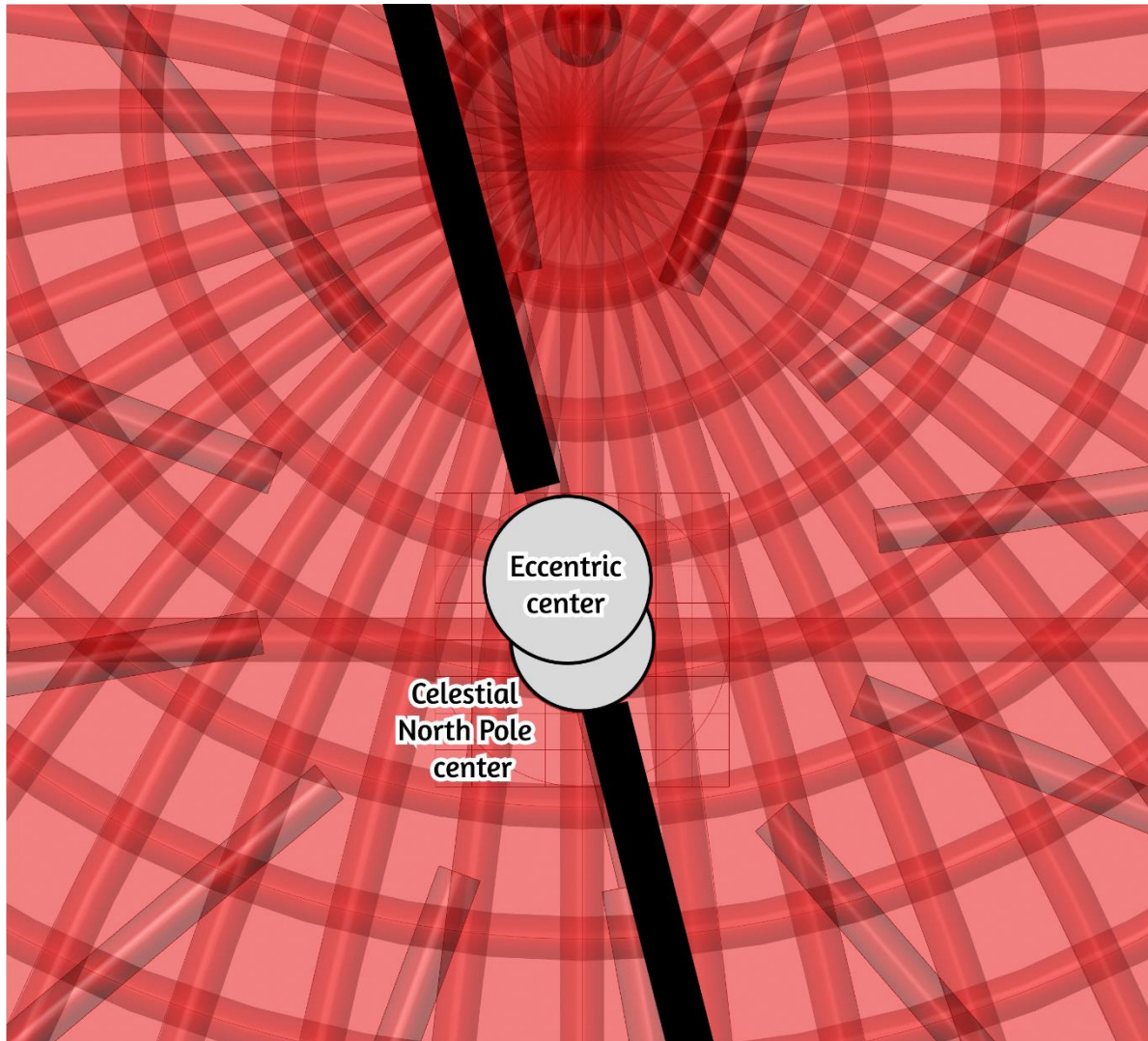
- Alkaid, Mizar, Alioth
- Megrez, Phecda, Merak, Dubhe



## Back of the astrolabe

The back of this 3D-printed astrolabe contains a diagram for converting the calendar date to a position of the sun in the Zodiac.

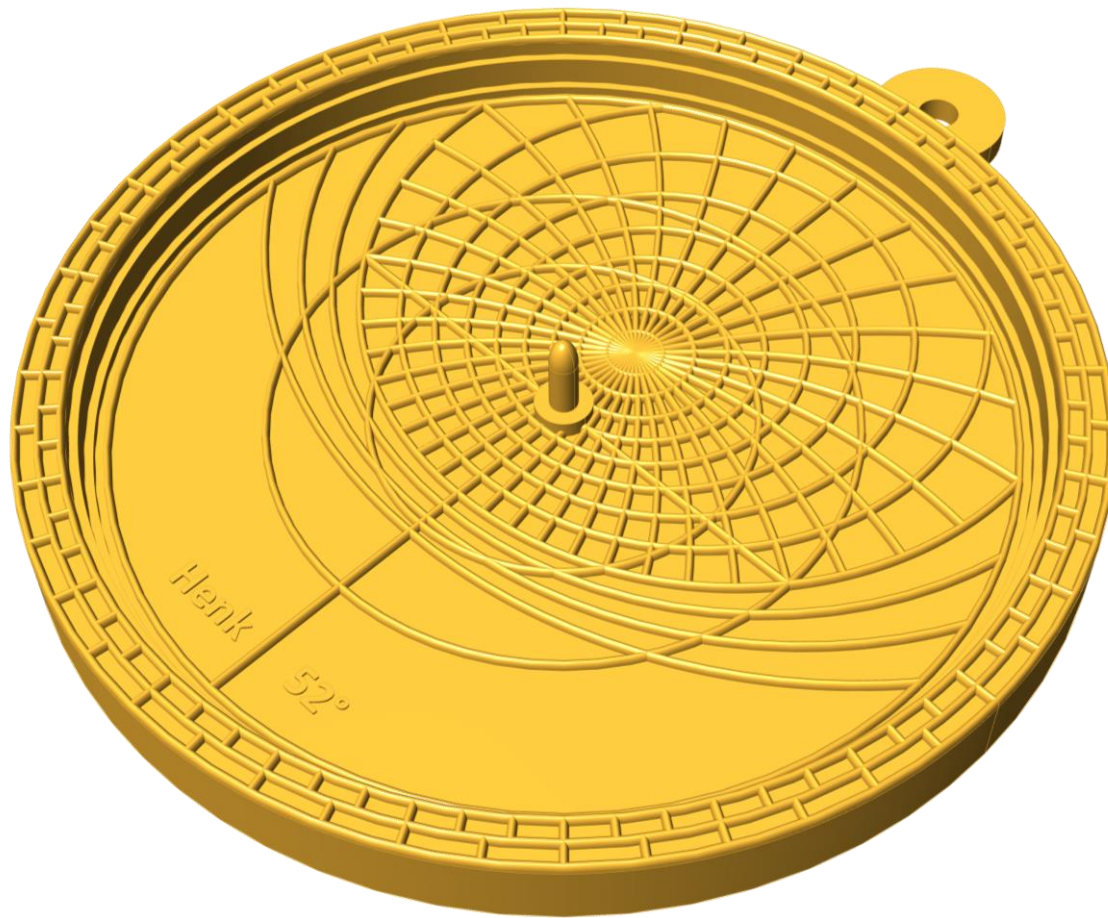
- Zodiac circle
  - Its center is Earth (the celestial North Pole)
  - Divided into the twelve zodiac signs,
  - Each sign covers  $30^\circ$ , divided into equal portions of five degrees.
  - First of Aries, or  $0^\circ$  Aries is marked by the sign of Aries
- Apogee and Perigee
  - At Apogee, the Sun is at the greatest distance from the Earth
  - At Perigee, the Sun is at the least distance
- Eccentric center
  - is the center of the calendar circle
  - has a small offset from the center of the zodiac circle
  - The eccentric center lies on the line from Apogee to Perigee
- Calendar circle
  - the orbit of the sun in its apparent yearly motion around the earth
  - divided into 365 days where each month has 30 or 31 days and February has 28 days



## Ruler at the back

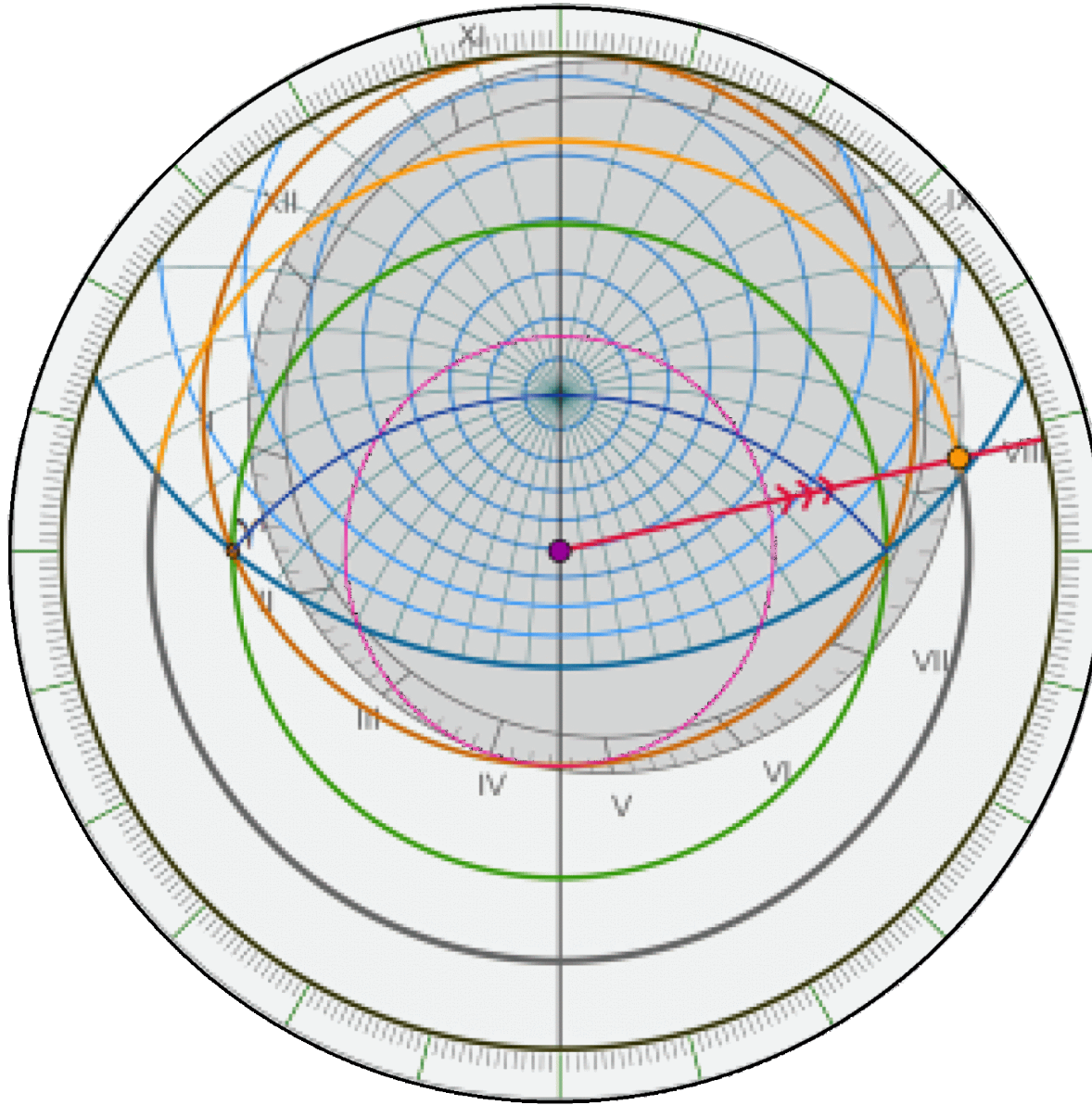
- The ruler at the back is at the eccentric center of the orbit of the sun in its apparent yearly motion around the earth





## Rhino / Grasshopper

Rhino / Grasshopper3D is my design program to develop a 3D astrolabe because of its parametric design capability. A FlashForge Adventure 3 printer produced the 3D astrolabe. For every specific latitude, for example, 52° for the Netherlands or 41° for Istanbul, Grasshopper delivers the proper design. More information is available on my website <https://www.fransvanschooten.nl/astrolabe.htm>.



## Animation

On my website, there is a GeoGebra animation that explains step-by-step how to draw all the lines, circles and curves. The animation can be configured for any time zone and latitude.

<https://www.fransvanschooten.nl/astrolabe.htm>

